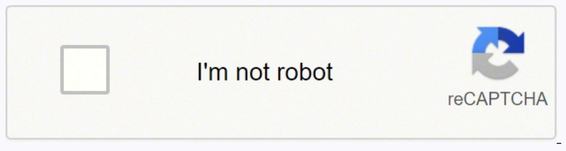

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Visually, the first derivative represents the slope of the curve P z (r, t, T), plotted against r, at the current interest rate level. This is in fact the bond that makes the bootstrap methodology fail at that maturity: The price seems too high compared to what the Nelson Siegel model – and in fact all of the other bonds around it – would imply. 2.2.1.3 Continuous Compounding. Figure 2.3 shows that the term structure of interest rates can be declining, with short-term spot rates higher than long-term spot rates. At the same time, it also defines a semi-annually compounded interest rate equal to r2 (t, T) = 4.43%. Similarly, if the house prices of that particular community decline, the collateral in the mortgage contract declines, and the local bank is then in a more risky position than before. Depending on how cash flows move with interest rates, this sensitivity can be larger or smaller than the maturity of the security itself. 1.2.2 Floating Rate Coupon Bonds The bonds issued by the U.S. government have a fixed coupon rate. Starting from the top left corner, we have: 1. This is typically the case when we consider maturities up to five years. Chapter 16 takes the model one step further, and discusses the issue of dynamic rebalancing and relative value trades. 13.3.2 Illustrative Example: Long-Term Interest Rate Options 13.3.3 How Many Simulations are Enough? 3.5 SUMMARY In this chapter we covered the following topics: EXERCISES 99 Table 3.5 Asset and Liabilities of a Financial Institution Assets Item Cash S.T. Loans M.T. Loans L.T. Loans Total Liabilities Amount Duration 100 300 500 1500 2400 0 0.8 3 12 Dollar Duration 0 240 1500 18000 19740 Item Deposits S.T. Debt M.T. Debt L.T. Debt Total Equity Amount Duration 600 400 400 400 1800 0 0.5 4 8 600 Dollar Duration 0 200 1600 3200 5000 14740 1. Why does a security that is in high demand entail a lower (special) repo rate? For this to work, we need to establish a fixed reference rate from which to subtract the floating rate. The chapter also includes an economic model of the term structure, which links the continuous time models illustrated in earlier chapters to the variation in expected inflation, and the compensation for risk that investors require to hold nominal securities when there is inflation. The large shift up in the interest rate was probably unexpected. The financial institution can purchase a set of securities that pays exactly $28,767 every six months. Solving Equation 2.30 for the two bonds, the yield to maturity of the c = 4.75 T-note and c = 8.875 T-bond are, respectively yc=4.750 = 3.7548%. Fixed Income Securities John Wiley and Sons, 2010 Valuation, Risk, and Risk Management This web site contains useful links for the readers of my book Fixed Income Securities: Valuation, Risk, and Risk Management published by John Wiley and Sons in January 2010. For instance, in Example 3.7 the VaR varies depending on whether we use the normal distribution approach or the historical distribution approach. 1 These data are obtained from the Web site accessed on August 22, 2006. Think again about the two extremes: 100% investment in long-term bonds loses money when interest rates go up, because bond prices decline when interest rates increase. 1.3.1 Federal Funds Rate Banks and other financial institutions must keep some amount of capital within the Federal Reserve. The institutional feature of lagging the coupon payment by six months allows for this cash flow and discount effect to exactly cancel each other out, leaving the value of the bond at 100 at any reset date. Definition 3.6 Let α be a percentile (e.g. 5%) and T a given horizon. However, an investment in a U.S. Treasury bond may not be safe in terms of its return on investment over a short period of time. The 2007 - 2009 crisis provides in fact an important example of market disruptions, and this book contains several examples and case studies discussing the risk and return of setting up and carrying out what appear to be arbitrage strategies. A similar precedure is used for the 1% expected shortfall calculation. For instance, if the financial institution was to keep 100% invested in the long term T-bond, it would stand to lose money about 50% of the time. The formula is as difficult as the Black and Scholes formula for options on stocks, so depending on how advanced the students are, they may or may not find the material challenging. Since the solution to bootstrap involves inverting the matrix C, we have a problem, as the matrix C must have an equal number of rows and columns to be inverted. Thus, the U.S. government debt market was the main reference for global fixed income markets. Case Studies The book contains several end-of-chapter case studies. Not only will fixed income markets be expanding in the future, as mentioned above, but in the past two years investors across the world dumped risky securities and purchased safe U.S. government securities, which pushed their prices up and their yields down. Similarly, if the interest accrues every month, then the correct answer to the same question is given by r = 4.89%, as ($100) × (1 + 4.89%/12) × . If we look at Equation 2.51, we see a regression equation of the type yi = α + n β j xij + εj j=1 where the data are yi = Pci (0, T i) and xij = Cij , and the regressors are β j = Z (0, Tj). As mentioned in point 3 in the above Subsection 3.2.8.1, the VaR measure does not say anything about the tails of the statistical distribution. 1. By Pietro Veronesi c 2010 John Wiley & Sons, Inc. Find an alternative decomposition of the same security, and compute the price. As a consequence, the market value of equity is $600 million, but with a dollar duration of $14.740 billion. Indeed, the extended Nelson Siegel model has been put forward to capture severe non-linearities in the shape of the term structure of interest rates, a situation that did not occur in 1993. I illustrate the use of Monte Carlo simulations for risk assessment both in examples, as well as in a case study at the end of chapter. The table corresponds to screen BTMM from Bloomberg terminals, and it is widely used by traders to quickly grasp the relative positions of bond prices and interest rates across markets. Three Month Rates Interest Rate (%) 8 6 4 2 0 Treasury Repo LIBOR 1992 1994 1996 1998 2000 2002 Source: Federal Reserve Board, British Bankers Association, Bloomberg. The cum-coupon price at t = 1.5 is then 100 plus the coupon, which equals 100 times the semi-annual rate determined at t = 1. The large estimate of the monthly standard deviation of interest rates σ dr = 0.00432 is mainly due to the large volatility in the 1970s and beginning of the 1980s. However, this book aims at clarifying two important issues: First, models have parameters and parameters need data to be estimated. These differences allow me to describe the various properties of the models. The investor decides to short the long-term bonds in each portfolio to raise the $50 million. (b) Is this difference a loss? N n n c (T1) c (T2) ... Such a strategy involves the choice of a portfolio of securities with the same present value and duration of the cash flow commitments to pay. (d) 6-year floating rate bond with a zero spread, paying semiannually (e) 3-year floating rate bond with a 35 basis point spread, paid semiannually (f) 4 1/4 year floating rate bond with 50 basis point spread, paid semiannually 2. With this link in mind, I wrote Chapter 20 in Part III in a way that does not need any of the more advanced material in the earlier chapters of Part III. Given that the financial market turmoil of 2007 - 2008 started in the mortgage backed securities markets, the chapter also describes some of the events during this period of time. What does the plot look like? Yes, because at time t = 0.5 the investor will know the interest rate. The price of the Treasury bill was $95.713. Using this method, each polynomial can be of low order and hence retain some stiffness, that is, a more stable curve. Consider an investor who is evaluating this bond. As the level of interest rates changes over time, banks, bond portfolio managers and corporations worry about the impact that the swings in interest rates have on the value of their assets and liabilities. The requested URL was not found on this server. As it can be seen, if we keep increasing n, the n− times compounded interest rate r n (t, t + 1) keeps decreasing, but at an increasingly lower rate. Panel A of Figure 3.1 plots the time series of yields from 1965 to 2005. From the law of one price (see Fact 1.1 14 Thanks to Francisco Javier Madrid for his help in putting this case together. 35 INTEREST RATES 2.2.1.1 Semi-annual Compounding Let's begin with an example: EXAMPLE 2.3 Let t = August 10, 2006, and let T = August 10, 2007 (one year later). Useful Links for Data or Other Fixed Income Market Sources Federal Reserve Statistical Releases: Updated weekly, this web site is a very useful source of various interest rates data. Figure 1.2 plots the stunning growth in interest rate derivatives markets. 3.1 THE VARIATION IN INTEREST RATES Interest rates change substantially over time. 1.3.3 LIBOR LIBOR stands for London Interbank Offer Rate. ⎠⎝ .. The economist Lars Svensson proposed an extension to the model, which is the one most widely adopted. As a consequence, the yield is lower than other regular Treasury notes and bonds, as the latter pay an income that is taxable according to investors' income tax rates. It is customary to annualize this amount, so that Annualized return on zero coupon bond = 1 Z(t, T) T 1−t −1 (2.29) This, of course, corresponds to the annually compounded yield on the zero coupon, as in Equation 2.6 for n = 1. An interesting fact arises by looking at the formula in Equation 2.42: Coupon payments are a combination of a fixed rate and a floating rate bond. For the expected shortfall, we take the average of all of the realizations below the 5% worst case. 32 BASICS OF FIXED INCOME SECURITIES Figure 2.1 Discount Factors 100 95 Discount Factor (%) 90 85 80 75 70 3 months 65 1 year 3 years 60 1955 1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010 Source: Center for Research in Security Prices (CRSP) plotted in Figure 2.1 with the expected annual inflation series in Figure 2.2 it appears that expected inflation is an important determinant of discount factors. For instance, Equation 3.17 shows that the duration of a fixed coupon bond equals the average time of payment times, which is a relatively simple formula to determine the sensitivity of a coupon bond to parallel shifts in the yield curve. In term of material, finally, my students also find it useful to connect the economic model discussed in Chapter 18 to the Vasicek model, discussed earlier, as well as to the evidence on expected return in Chapter 7, as they see the connections between risk, risk aversion, return, market price of risk, and, ultimately, pricing. Panel B of Figure 3.1 plots the simple average of yields graphed in Panel A. We use these models also to price standard derivatives, such as caps, floors, swaps and swaptions. On the other hand, the U.S. Treasury used Congress-approved funds to bail out a number of financial institutions, while the Federal Deposit Insurance Corporation (FDIC) extended guarantees on the short-term debt of banks in risk of default. Between t and T the trader (who is long the bond) earns the interest that accrues on the bond. Compute the price, the yield and the continuously compounded yield for the following Treasury bills. For instance, if an arbitrageur finds two securities that pay exactly $100 in six months, but one trades at P1 = $97 and the other at P 2 = $98, then an arbitrageur can apply the trader's motto "buy low and sell high," and purchase 1 million units of Security 1 at $97 and sell 1 million units of Security 2 at $98, realizing an inflow of $1 million. In particular, the chapter illustrates the notion of options as financial insurance contracts, which pay only if some particular event takes place. When the repo rate is zero, however, the cost for a trader to fail to deliver the bond is very small, as the trader may keep the bond itself if this bond is particularly valuable. Although this is a topic of a later chapter, it is useful to provide here the most obvious, and intuitive, reason. (a) 4-week with 3.48% discount (December 12, 2005) (b) 4-week with 0.13% discount (November 6, 2008) (c) 3-month with 4.93% discount (July 10, 2006) (d) 3-month with 4.76% discount (May 8, 2007) 58 BASICS OF FIXED INCOME SECURITIES (e) 3-month with 0.48% discount (November 4, 2008) (f) 6-month with 4.72% discount (April 21, 2006) (g) 6-month with 4.75% discount (June 6, 2007) (h) 6-month with 0.89% discount (November 11, 2008) (i) 1-year with 1.73% discount (September 30, 2008) (j) 1-year with 1.19% discount (November 5, 2008) 3. For instance, the 3-year discount factor is as low as 0.6267 in August 1981, and as high as 0.95 in June 1954 and in June 2003. If today is not t = 0, but t = 0.25, how do we value the floating rate bond? How do all these market rates move together? Through the use of a mix of structure notes and leverage, Orange County's portfolio stood to make subtantial above market returns were the interest rate not to increase in the near future. 22 AN INTRODUCTION TO FIXED INCOME MARKETS in it. The bottom line of this discussion is that fixed income markets are very large and still growing. While the U.S. is (still) very unlikely to default, default risk is but one of the risks that affect the value of Treasury bonds, and an understanding of the possible losses from an investment in "safe" Treasury bonds is key, especially in an environment of low interest rates such as the current one. 9 We use fair prices, i.e., prices obtained from the same discount curve Z(0, T), to better illustrate the concept of yield to maturity and its relation to a bond coupon rate. Table 1.1 provides a snapshot of the sizes of fixed income markets as of December 2008. Give a quote d from a Treasury dealer, we can compute the price of the Treasury bill by solving for P bill (t, T) in Equation 2.35: n Pbill (t, T) = 100 × 1 − ×d (2.36) 360 2.4.4.2 Treasury Coupon Notes and Bonds. There are three key elements in Definition 1.1: The trade (1) costs nothing; (2) yields positive profits with certainty; and (3) the profits arrive within a known time. For all practical purposes, however, daily compounding – the standard for bank accounts – closely matches the continuous compounding, as we see in the next example. 3.2.9 Duration and Expected Shortfall Some of the problems with VaR can be solved by using a different measure of risk, called the expected shortfall. We can answer this question by computing the duration of the portfolio: The 5-year and 10-year strips have duration of 5 and 10, respectively. Once again, there are numerous reasons that contribute to the variation of both interest rates and term spreads, such as fluctuations in expected inflation, expected economic growth, and risk attitude of investors. PART I: BASICS. The final chapter of Part I is Chapter 8, which contains a discussion of the mortgage backed securities (MBS) market, its main players, and the securitization process. The issuance market price was $97.477 for $100 of face value.1 That is, on August 10, 2006, investors were willing to buy for $97.477 a government security that would pay $100 on February 8, 2007. Sometimes the analysis require students to gather data from other sources available on public web sites, such as the LIBOR fixes available at the British Bankers' Association Web site, or the swap rates, available at the Federal Reserve Web site. Indeed, from Fact 3.7 the 99% expected shortfall in the case of the normal distribution is only $680 million, which is much smaller than the $990 million expected shortfall obtained under the historical distribution approach. Treasury bonds are the only type of security she would consider, given her age. On-the-run Treasury securities tend to trade at a premium compared to similar off-the-run Treasury securities, which tend to be less liquid than the on-the-run securities. The quote represents the price at which delivery will take place in the future. Instead, a definition in terms of yield to maturity hinges on the notion of yield to maturity itself, which, as discussed in Chapter 2 (Section 2.4.3) has some issues, such as the fact that it is bond specific, it depends on the coupon rate, and so on. However, such models may be too simplistic to design an arbitrage strategy. That is, the coupon on the bond is: c(t) = 15% − r1 (t − 1) (2.42) where r1 (t − 1) denotes the annually compounded rate at time t − 1, and we adopt the usual convention according to which the cash flow at time t, c(t), depends on the interest rate one period earlier, namely t − 1 in our case as payments are annual. Unless the term structure of interest rates is exactly flat, the yields at various maturities are different, and will not coincide with the yield to maturity y. Since dP is the change in the price of the security, in dollars, the name dollar duration follows. Actual/360: Each month has the right number of days according to the calendar, but there are only 360 days in a year. I also thank Chetan Dandavate and Camilo Echeverri for pointing out some important typos in the manuscript. In this turmoil, the U.S. government has taken the center stage: On the one hand, the Federal Reserve decreased its reference short-term interest rate, the Federal Funds target rate, to almost zero, and acted swiftly to set up lending facilities to provide liquidity to the financial system. I use this chapter to discuss both the relative pricing of different fixed income instruments, the notion of risk premium of a fixed income security, as well as the popular pricing methodology called risk neutral pricing. These concepts are so important in modern financial markets that I decided to present this material in isolation from the previous chapters in Part III, so that the material in this chapter stands alone, and can also be used as a concluding chapter after Chapter 11. This introductory chapter to mortgage backed securities also contains a discussion of the main measures of prepayment speed, as well as their impact on the pricing and risk exposure of several MBS, such as simple pass throughs, collateralized mortgage obligations, and principal only and interest only strips. I focus on the Vasicek model, a model that is relatively simple but also realistic, and provide several examples on the pricing of real-world securities. Copyright 3 4 AN INTRODUCTION TO FIXED INCOME MARKETS Table 1.1 The Size of Fixed Income Markets: December 2008 Market Market Value (billion of dollars) U.S. Treasury Debt U.S. Municipal Debt U.S. Federal Agency Securities U.S. Money Market 5,912.2 2,690.1 3,247.4 3,791.1 Mortgage Backed Securities Asset-Backed Securities 8,897.3 2,671.8 OTC Interest Rate Swaps OTC Interest Rate Forwards OTC Interest Rate Options Exchange Traded Futures Exchange Traded Options 16,572.85 153.19 1,694.22 U.S. Corporate Debt Credit Derivatives 6,280.6 5,651 Notional (billion of dollars) 328,114.49 39,262.24 51,301.37 19,271.05 35,161.34 41,868 Source: Securities Industry and Financial Market Association (SIFMA) and Bank for International Settlements (BIS). 6 AN INTRODUCTION TO FIXED INCOME MARKETS Figure 1.2 The Growth in Derivatives Markets: Notional 350 Swaps 300 Trillion of Dollars 250 OTC and Exchange Traded Interest Rate Options 200 150 Forward Rate Agreements and Futures 100 50 0 1986 1988 1990 1992 1994 1996 1998 2000 2002 2004 2006 2008 Source: SIFMA and Bank for International Settlement 1.1.1 The Complexity of Fixed Income Markets The previous section illustrates the growth in size of fixed income markets. The VaR is that number such that: P rob (LT > V aR) = α% (3.27) For instance, a $100 million bond portfolio may have a 95%, 1-month VaR of $3 million. Figure 1.6 plots the time series of the one month and the three month T-Bill, Repo and LIBOR rates from May 1991 to April 2008. Bootstrap: This procedure is for computing discount factors at various maturities from data on coupon notes and bonds. 96 Asset-Liability Management 97 Summary 98 Exercises 99 Case Study: The 1994 Bankruptcy of Orange County 103 3.7.1 Benchmark: What if Orange County was Invested in Zero Coupon Bonds Only? These are strategies that allow a trader to hedge a contingent payoff in the future by using a portfolio of other fixed income securities, and understanding them is at the heart of no arbitrage pricing. + + 100 × Z (t, Tn) 2 n c × 100 Z(t, Ti) + 100 × Z(t, Tn) × 2 (2.13) i=1 c Pz (t, Ti) + Pz (t, Tn) × 2 n = (2.14) i=1 The subscript "c" is a mnemonic device for "coupon" in coupon bond. How do we pick the bonds to bootstrap out the term structure of interest rates? 11 Each coupon date is also called reset date as it is the time when the new coupon is reset (according to the formula). Because the trader has to pay the repo rate during this period, setting up the repo transaction tends to generate a positive or negative stream of payments, depending on whether the interest earned on the bond is above or below the repo interest. For instance, it can purchase 60 zero coupon bonds, each with a $28,767 face value, and with maturities of 6 months, 1 year, 1.5 years, and so on up to 30 years. It turns out however that on February 15, 2008, the 9.5-year T-bond with c = 8.875 was actually trading at 140.0781, about 1% less than its fair price computed in Equation 2.32. Instead, 48 BASICS OF FIXED INCOME SECURITIES Equation 2.15 is the one defining the price of the bond from the discount factors Z(t, T). A fixed income book cannot sidestep the central banks' influence on fixed income securities. Par value is the bond's principal amount. In addition, interest rate floaters are issued by financial institutionsand corporations, as well as government agencies, such as the government mortgage companies Ginnie Mae, Freddie Mac, and Fannie Mae, within their collateralized mortgage obligations programs. Similarly, a 100% ASSET-LIABILITY MANAGEMENT 97 Table 3.4 The Duration Mismatch Commercial Banks Insurance Companies Pension Plans Corporations Assets Liabilities Long-term loans (High D) Short-Term T-Bonds (Low D) Medium-Term T-Bonds (Low D) Long-Term Receivables (High D) Deposits (Low D) Long-Term Commitments (High D) Long-Term Commitments (High D) Floating Rate Bonds (Low D) investment in cash loses money when interest rates go down. Three years from December 31, 1993 corresponds to the maturity date December 31, 1996. Finally, the chapter introduces the concept of spot rate duration, which is a concept of duration analogous to the one introduced in Chapter 3, but for securities defined on binomial trees. Plot the resulting bond price versus the yield to maturity. In all three cases, the term structure of interest rates is increasing, but it is clear that it is lower and steeper for the first date, while it is higher and flatter for third date. (b) Compute the price of the coupon bond for yields ranging between 1% and 15%. The debt securities issued by these mortgage giants are not as simple as U.S. Treasury securities, as they have a number of additional features, such as embedded options of various kinds, that make their valuation and risk assessment difficult. Using the semi-annually compounded yield curve in Table 2.4, price the following securities: (a) 5-year zero coupon bond (b) 7-year coupon bond paying 15% semiannually (c) 4-year coupon bond paying 7% quarterly (d) 3 1/4-year coupon bond paying 9% semiannually (e) 4-year floating rate bond with zero spread and semiannual payments (f) 2 1/2-year floating rate bond with zero spread and annual payments (g) 5 1/2-year floating rate bond with 35 basis point spread with quarterly payments (h) 7 1/4-year floating rate bond with 40 basis point spread with semiannual payments 5. To make this point clearer, we discuss a famous case involving a special swap between Procter & Gamble and the investment bank Bankers Trust. In well-functioning markets such arbitrage opportunities cannot last for long. The duration of the floating rate bond is then Duration of floating rate bond at t = = = = 1 d PF R (3.15) PF R (t, T) d r 1 d Z(t, Ti+1) r2 (Ti) − × 100 × 1 + PF R (t, T) dr 2 r2 (Ti) 1 [−(Ti+1 − t)] × Z(t, Ti+1) × 100 × 1 + − PF R (t, T) 2 (3.16) Ti − t DF R = − where the last equality stems from using again Equation 3.14. On the other hand, a 100% investment in cash only (Panel D) also is not appropriate, as the strategy loses money again about 50% of the time. 9. These chapters discuss several examples in which models are applied to real world securities, and draw some distinctions among them. For convenience, we refer to both types as coupon bonds. The slight decrease in this market size visible at the end of the sample, in 2008, is indeed a reflection of the decline in the housing market and the U.S. recession that started in January 2007. Instead, assuming that the 30-year bond sells at par and that the coupon rate is 4%, strategy 2 provides a certain $40,000 per year for all 30 years. In this case, we resort to the dollar duration: Definition 3.4 The dollar duration D $ of a security P is defined by Dollar duration = D P$ = − dP dr (3.21) That is, the dollar duration corresponds to the (negative of the) sensitivity of the price P to changes in the level of interest rate r. As mentioned, such examples are always based on real data and therefore on real situations. That is, investing in U.S. Treasury securities is considered "safe", as the government will repay its debt to investor. Eventually, it converges to a number, namely, 4.879%. 3.7 CASE STUDY: THE 1994 BANKRUPTCY OF ORANGE COUNTY As discussed in Section 3.1.2, in 1994 Orange County lost $1.6 billion out of a portfolio of $7.5 billion in assets as a result of an unexpected increase in interest rates, from 3% to 5.7%.6 Figure 3.6 shows the sudden steep increase in the level of interest rates in 1994. A 1-year bond with 13.13% coupon priced at $106.00 (issued 4/2/1981) Compute the prices for these bonds with the discounts you found. Although neither figure can be compared directly to the U.S. debt market, for a number of reasons discussed in Chapter 5, the sizes of these markets once again demonstrate that the U.S. debt market has been eclipsed by other types of securities. Consider first Example 2.13. As mentioned, the repo market has grown steadily over the years. (b) How do the yields to maturity compare to each other? This decomposition results in the equality Floating coupon with spread s = Floating coupon with zero spread + Fixed coupon s We can then value independently each component on the right-hand side, as we already know how to value a floating coupon bond with zero spread (see previous section) and a sequence of fixed coupon payments equal to s. However, we can use computers to simulate interest paths on the tree itself, and therefore obtain the prices and hedge ratios of these securities by simulation. It generates a positive profit at initiation, and it has a sure nonnegative payoff by a certain date in the future. An interesting outcome of such penalty on a fail to deliver is that on May 1, 2009, the repo rate for some Treasury securities that were on special became negative.5 How is it possible that an interest rate is negative? We will see that different models may yield different answers about the value of the same derivative security even when PREFACE xxiii using the same data to estimate their parameters. For instance, the U.S. government debt in marketable securities stood at about $6 trillion at the end of 2008, around 40% of the U.S. Gross Domestic Product (GDP), and the congressional budget office (CBO) predicts an additional $1.8 trillion U.S. deficit for 2009. Other key rates, such as the prime. Let P 1 and P2 be the prices of these two securities, respectively. An investment bank observing the rates in Table 1.7 may offer the following deal to the two firms. At that time, investment banks would also reap substantial profits from relatively large spreads. A case study at the end of the chapter further illustrates these concepts within the pricing of inverse floaters, which are popular fixed income securities yielding higher-than-market returns if interest rates decline. The price was $98.739 for $100 of face value. And so on until t = 0. For the time being assume that we know with certainty that the short rate is always below 15%. Table 1.1 shows that indeed the options' market is quite larger than the futures and forward markets. Table 2.8 The Price of a 15% Fixed Coupon Bond Date Cash Flow Discount Z(0, T) Discounted Cash Flow 19931231 19941231 19951231 19961231 0.15 0.15 1.15 0.9642 0.9193 0.8745 0.1446 0.1379 1.0057 Sum Price (×100) 1.2883 128.83 3. The question is then: How much lower is Z(t, T 3), say, compared to Z(t, T2) or Z(t, T1)? However, in many interesting cases involving no arbitrage strategies, the security or the portfolio may have a value of exactly zero. 91 INTEREST RATE RISK MANAGEMENT EXAMPLE 3.9 Ms. Caselli retired at the age of 60, with $1,000,000 in her retirement account. Thus, the value at time t = 0 is Value bond at 0 = Present value of (100 + c(0.5)) = 100 + 1 = 100 1 + .02/2 The result that at time t = 0.5 the ex-coupon bond price of this floating rate bond is always equal to 100 may appear puzzling, but it is actually intuitive. xxviii PREFACE Data The book relies heavily on real-world securities data. In one year, the original investment will pay $105.0625, as we obtained earlier, and thus the rate of return is 5.0625%>5%. We can also represent the value of the coupon bond by using the semi-annual interest rate r2 (t, Ti), where Ti , i = 1, ..., n, are the coupon payment dates. Chapter 3 contains the basics of risk management: The chapter introduces the concept of duration, and its use to design effective hedging strategies, as in asset-liability management. However, such a position at maturity entails that we receive $100 from the long position and we must pay $200 from the short position. Chapter 10 expands the concepts of one-period binomial trees to multiple periods, and discusses the issue of dinamic hedging, the standard methodology of hedging a risk exposure by rebalancing the portfolio over time as the interest rate changes. Definition 2.5 Let t be a given date. The semi-annually compounded yield to maturity, or internal rate of return, is defined as the constant rate y that makes the discounted present value of the bond future cash flows equal to its price. (c) Does the conclusion arrived at in Exercise 2 stand? Given μ P = −.0033 and σ P = 2.0767, we obtain (Normal disttribution approach): 95% ES = $4.2871 mil; 99% ES = $5.5374 mil (3.35) The numbers are quite different for the case in which the historical distribution approach is used. Thus, an increase in coupon rate implies an overall lower sensitivity to changes in discount rates. The duration of a zero coupon bond equals its time to maturity, thus D zero = 3. Returning to Example 2.5, we can verify Equation 2.8 by taking the natural logarithm of Z(t, T) = $100/$105 = .952381 and thus obtaining r(t, T) = − ln(Z(t, t + 1)) = 4.879% 1 38 BASICS OF FIXED INCOME SECURITIES 2.2.2 The Relation between Discounts Factors and Interest Rates The previous formulas shows that given a discount factor between t and T , Z(t, T), we can define interest rates of any compounding frequency by using Equations 2.2, 2.5, or 2.7. This fact implies that we can move from one compounding frequency to another by using the equalities implicit in these equations. 2 2 12 Ex-coupon means that the price does not incorporate the coupon that is paid on that particular day. Using the same steps as in Table 3.8 but with coupon rate c = 25% we find that the duration of the coupon bond in this case is D c = 2.5448. 30-years mortgage backed securities; 12. Garbade, "When the Back Office Moved to the Front Burner: Settlement Fails in the Treasury Market after 9/11," Federal Reserve Bank of New York Economic Policy Review, November 2002. This large change in the term structure of interest rates may have a devastating effect on the value of portfolios heavily invested in fixed income instruments. Before we investigate how no arbitrage and the law of one price allow us to study the valuation, risk and risk management practices of fixed income instruments, let's take a closer look at the fixed income markets, using the entries in Table 1.2 as a guide. The reason is that standard adjustable rate mortgages contain a provision stating that the maximum rate the homeowner will have to pay over the life of the mortgage is capped at some level. 2 Second, in 2008 the U.S. government took upon its shoulders the two mortgage giants Freddie Mac and Fannie Mae, and therefore their trillion dollars worth of debt can now be considered as safe (or as risky) as U.S. government securities, further expanding the effective size of U.S. government debt. There are securities that cannot be easily priced on binomial trees because their payoff at maturity may depend on a particular path of interest rates. If a bond is purchased between coupon dates, the buyer is only entitled to the portion of the coupon that accrues between the purchase date and the next coupon date. Recall that the annually compounded interest rate is r 1 (t, t + 1) = 5%, the semi-annually compounded interest rate is r 2 (t, t + 1) = 4.939%, and the monthly compounded interest rate is r 12 (t, t + 1) = 4.889%. CASE STUDY: THE 1994 BANKRUPTCY OF ORANGE COUNTY 105 3.7.2 The Risk in Leverage Orange County's portfolio, however, was not only invested in Treasury securities. It may happen that due to lack of liquidity or trading, some arbitrage opportunities may be detectable in the relative pricing of zero coupon bonds, such as STRIPS, and coupon bonds. In general, however, it will not be possible to find parameter values that price all of the bonds exactly, because of staleness in the data, lack of liquidity, or lack of degrees of freedom in the Nelson Siegel model (we only have four parameters, after all). For instance, in Part I exercises require spreadsheets to compute the prices of complicated securities from simpler ones, or their duration and convexity. From Fact 3.6, a normal distribution assumptin on dr translates into a normal distribution on dP . The world of fixed income markets has being becoming increasingly more complex, with debt securities that have the most varied payoff structures, and fixed income derivatives that are growing in sheer size and complexity. Unlike the examples, which are tightly focused on the particular issue just being discussed in the chapter, a case study describes a situation and then carries out the whole analysis, although of course still within the topic discussed in the chapter. I use data to illustrate the examples in the body of the textbook as well as to discuss the case studies at the end of chapters. In reality, the exact split depends on the relative contractual strength of each firm: Firms with higher creditworthiness would tend to get a higher coupon. In contrast, each column j describes all the cash flows that occur on that particular maturity Tj from the n bonds. Tn be the floating rate reset dates and let the current date t be between time Ti and Ti+1 : Ti < t < Ti+1 . I apply the concepts of Brownian motions and Ito's lemma in Chapter 15 to illustrate the notion of no arbitrage, and obtain the fundamental pricing equation, an equation that we can use to compute the price of any fixed income derivative. Table 1.2 reports a snapshot of rates in the U.S. fixed income markets on September 18, 2007. Thus, Equation 2.13 should hold "most of the time". Figure 2.6 shows the results of applying the extended Nelson Siegel model to the data in Table 2.6. The parameter estimates are θ0 = 0.0687, θ1 = −0.0422, θ2 = −0.2399, θ3 = 0.2116, λ1 = 0.9652, and λ2 = 0.8825. 8 Column 7 shows the discount curve Z(0, T) obtained from the bootstrap procedure discussed in Section 2.4.2, and Column 8 reports the continuously compounded spot rate curve r(0, T). The discount channel: If interest rates go up, prices of zero coupon bonds fall as future cash flows are worth less in today's money. In modern financial markets computers are just a necessary part of the analysis toolbox. 2.3 THE TERM STRUCTURE OF INTEREST RATES In the previous sections we noted that the primitive of our analysis is the discount factor, from which we define interest rates of various compounding frequencies. [12 times] . This chapter concludes the second part of the book. In percentage, this corresponds to a 24%decline in market value of equity. c2 (Tn) ⎟ ⎟ ⎟ C =⎟ . Zero coupon bonds: These are securities that pay only one given amount (par) at maturity. We may refer to this average generically as the level of interest rates. In later chapters we use modern financial concepts to precisely define the expected return on an investment during a given period, as well as the no arbitrage restrictions that must exist across bonds. Because depositors could choose where to put their money, banks were forced to offer high deposit rates, otherwise depositors would withdraw their deposits and invest in other securities, such as Treasuries. Repurchase agreements and the repo rate: In collateralized borrowing between two counterparties, the repo rate is the borrowing or lending rate within a repurcahse agreement. THE GOVERNMENT DEBT MARKETS 13 Table 1.5 Stripped Coupon Interest on September 25, 2008 Maturity Year Month Day Bid Ask 2008 11 15 99.898 99.918 2009 2 15 99.478 99.498 2009 5 15 98.979 98.999 2009 8 15 98.473 98.493 2009 11 15 97.982 98.002 2010 2 15 97.487 97.507 2010 5 15 96.879 96.899 2010 8 15 96.294 96.314 2010 11 15 95.722 95.742 2011 2 15 94.83 94.85 2011 5 15 94.304 94.324 2011 8 15 93.274 93.294 2011 11 15 92.957 92.977 2012 2 15 91.072 91.092 2012 5 15 90.705 90.725 2012 8 15 89.274 89.294 2012 11 15 88.498 88.518 2013 2 15 87.478 87.498 2013 5 15 86.684 86.704 2013 8 15 85.988 86.008 2013 11 15 85.014 85.034 2014 2 15 83.999 84.019 2014 5 15 83.172 83.192 2014 8 15 82.185 82.205 2014 11 15 81.257 81.277 2015 2 15 79.706 79.726 2015 5 15 78.898 78.918 2015 8 15 77.972 77.992 2015 11 15 76.772 76.792 2016 2 15 75.885 75.905 2016 5 15 74.437 74.457 2016 8 15 73.593 73.613 2016 11 15 72.086 72.106 2017 2 15 71.16 71.18 2017 5 15 70.144 70.164 2017 8 15 69.036 69.056 2017 11 15 68.213 68.233 2018 2 15 67.643 67.663 2018 5 15 66.816 66.836 2018 8 15 65.674 65.694 2018 11 15 64.851 64.871 2019 2 15 63.626 63.646 2019 5 15 62.904 62.924 2019 8 15 61.826 61.846 2019 11 15 61.081 61.101 2020 2 15 60.194 60.214 2020 5 15 59.29 59.31 2020 8 15 58.475 58.495 2020 11 15 57.716 57.736 2021 2 15 56.876 56.896 2021 5 15 56.128 56.148 2021 8 15 55.368 55.388 2021 11 15 54.649 54.669 2022 2 15 53.989 54.009 2022 5 15 53.282 53.302 2022 8 15 52.599 52.619 2022 11 15 51.869 51.889 2023 2 15 51.144 51.164 2023 5 15 50.606 50.626 Source: The Wall Street Journal. Some content that appears in print, however, may not be available in electronic format. As we can see large increases and decreases are not very likely, but they do occur occasionally. Indeed, to some extent, mentioning only an interest rate level is an incomplete description of the rate of return of an investment, or the cost of a loan or mortgage. With these data we can get a sense of the composition of the Orange County portfolio. Table 1.2 does not report quotes of forward contracts, although we see from Table 1.1 that forward contracts make up a sizable share of the fixed income market. Therefore, the duration approximation method is internally incosistent. The repo interest is computed as the repo rate agreed at time t times the time between t and T . Immunization is preferred over cash flow matching as it allows the institution to choose bonds that have favorable properties in terms of liquidity and transaction costs. Part II of the book, and especially Chapter 11, discusses the pricing and hedging of plain vanilla derivatives, such as caps, floors, swaps, and swaptions. Total liabilities are $1.8 billion with a dollar duration of only $5 billion. The book also highlights the fact that most of the analysis of fixed income securities must rely on some models of the term structure, that is, some particular assumptions about the movement of yields through time. 70% in T−Bond 800 1200 Number of Events Number of Events 1000 600 400 200 800 600 400 200 0 −3,000,000 −1,500,000 Dollars 0 −200,000 0 0 Dollars C. See also the article by Michael J. U.S. Treasury market: The U.S. issues four types of securities: short-maturityT-bills, medium-maturity T-notes, long-maturity T-bonds, and TIPS, the inflation-protected securities. Structured securities, such us leveraged inverse floaters, contain additional risks that the risk manager must be aware of. The U.S. government, as with most governments, needs to borrow money from investors to finance its expenses. • If the interest rate r 2 (0, 5) declines, the discount rate declines. We can do this by using the concept of duration introduced in Section 3.2. Recall that duration is defined as the (negative of the) sensitivity of a security, or a portfolio, to parallel shifts in the term structure of interest rates. Indeed, the present value of the fixed T sequence of payments equal to s is t=0.5 s × Z(0, t). Conclusion To conclude this brief introduction to the book, let me mention that I truly hope that this book will encourage readers and students to analyze fixed income markets in a very systematic way, always looking for the reason why some events occur, some trades seem possible, or some models may or may not work. Formula 2.14 shows that the coupon bond can be considered as a portfolio of zero coupon bonds. In wellfunctioning markets, any risk embedded in fixed income securities should be compensated for by a risk premium on its rate of return, either through a high coupon or a low purchasing price. This value is much higher than the discount factor with the same time to maturity (six months) two years later, on August 10, 2006, which we found equal to 0.97477. Copyright 73 74 BASICS OF INTEREST RATE RISK MANAGEMENT they were all relatively high in the late 1970s and early 1980s, and they were all relatively low in the late 1990s. Coupon notes and bonds. In addition, the futures market is often used by market participants to gauge the market expectation about future movement in interest rates. If for instance r4 (0.25, 0.5) = 1%, the value of the bond at 0.25 is $100.7481. For every future coupon date, we have: Firm A Pays : Firm B Pays : (LIBOR + 3%) + 11% − LIBOR = 14% to market swap deal 12% −11% + LIBOR = LIBOR + 1% Figure 1.8 shows the cash flows in every period. Sometimes we in fact have too many maturities and sometimes we do not have enough maturities available to carry out the bootstrap procedure. Assume today is May 15, 2000, which means you may use the yield curve presented in Table 3.6 Portfolio A • 40% invested in 4 1/4-year bonds paying 5% semiannually • 25% invested in 7-year bonds paying 2.5% semiannually • 20% invested in 1 3/4-year floating rate bonds with a 30 basis point spread, paying semiannually • 10% invested in 1-year zero coupon bonds • 5% invested in 2-year bonds paying 3% quarterly Portfolio B • • • • 40% invested in 7-year bonds paying 10% semiannually 25% invested in 4 1/4-year bonds paying 3% quarterly 20% invested in 90-day zero coupon bonds 10% invested in 2-year floating rate bonds with zero spread, paying semiannually • 5% invested in 1 1/2 -year bonds paying 6% semiannually 3. The 36 BASICS OF FIXED INCOME SECURITIES semi-annually compounded interest rate can be computed from Z(t, T) = 0.95713 by solving for r 2 (t, T) in Equation 2.2: 1 1 r2 (t, T) = 2 × =2× = 4.43% (2.3) 1 − 1 1 − 1 0.95713 2 Z (t, T) 2 Fact 2.5 Let Z(t, T) be the discount factor between dates t and T . Chapter 14 introduces the notions of Brownian motion, differential equations and Ito's lemma. For instance, in Table 1.2, the heading "30 Y MBS" reports the prices of popular mortgage-backed securities, those issued by Ginnie Mae (GNMN 6.0), Freddie Mac (GOLD 6.0) and Fannie Mae (FNMA 6), the three largest players in the mortgage-backed securities market. What is the rate of interest on this security? These tools are very important to uncover the often hidden risks in some structured interest rate securities. You decided to hedge your portfolio a 3-year coupon bond paying 4% on a semiannual basis. Discount factors: The discount factor is the value today of one dollar in the future. These case studies apply the concepts developed in the chapter to more complex real-world situations. We should always keep in mind that the two quantities are equivalent. Students tend to enjoy the comparison across models, and why some models work in some interest rate environemnts and not in some others. To any extent, we should consider the swap market a primary market whose value is driven by investors', speculators' and end users' fluctuating demands. Essentially, all fixed income securities are linked to each other by the variation of interest rates, and therefore they move in a highly correlated fashion. EXAMPLE 3.11 Consider a hypothetical financial institution mainly engaged in making long-term loans. Namely, we can compute N i, NS model 2 − Pci, data J(θ 0 , θ1 , θ 2 , λ) = Pc (2.55) i=1 The Nelson Siegel model works perfectly if the model prices equal the data, i.e., if for every i = 1, ..., N we have Pci, NS model = Pci, data . Similarly, Firm B pays LIBOR +1% instead of LIBOR +2%, which was its market rate. A pure arbitrage trade consists in taking positions that generate, magically to some extent, always nonnegative cash flows, and with certainty, some positive cash flow. In the above example, the two dates are t = August 10, 2006 and T = February 08, 2007. One important problem is that financial institutions have very complex asset composition. The implication of this mismatch is that a parallel upward shift in interest rates of 1% generates a decline in assets far greater than in liabilities, implying an equity decline of $147.4 million. Part III: Continuous Time Models Part III covers more advanced term structure models that rely on continuous time mathematics. We can start from the left-hand side of the distribution, and move rightward until we count 5% of the observations. Using the normal distribution approach and assuming that monthly changes in interest rates are independent and identically distributed – a strong assumption as there is some predictability in yields, as discussed in Chapter 7 – the annualization can be√performed by multiplying the mean μ P by 6 and the monthly standard deviation σ P by 6. The notion of discount factors is at the heart of fixed income securities. 2.4.1 From Zero Coupon Bonds to Coupon Bonds In this section we establish a relation between the prices of zero coupon bonds and coupon bonds. (a) How many bonds of each kind does the investor have to short? Table 3.8 performs the computation, obtaining D F ixed = 2.6684. Indeed, in many instances it is no longer clear what a real "derivative" security is. Students at the end of the course will have the tools to tackle the proper analysis of real-world securities, assess their risk, and perform Monte Carlo simulations (on binomial trees) to value complex securities. As we shall see, formulas and derivations are much simpler under the assumption that the interest on an investment accrues infinitely frequently. The chapter also illustrate some drawbacks of using simple models. EXAMPLE 3.6 Dollar Duration of a Long-Short Strategy Let the term structure of interest rates be flat at 4% (semi-annually compounded). The reasoning is the same, and we work backwards. This can be seen, once again, from the prices of U.S. Treasury securities. Unlike futures, forward contracts are not traded on regulated exchanges but only on the over-the-counter market. The difference can be large. 1.4.2 What if the T-bond Is Not Delivered? However, it is also clear that they do not move up and down by the same amounts. Note that the rates available to B are always lower than the ones available to A, reflecting a difference in credit risk. Definition 2.4 The term spread, or slope, is the difference between long-term interest rates (e.g. 10-year rate) and the short-term interest rates (e.g. 3-month rate). However, the greater flexibility offered by the continuous time model enables me to discuss many more models which are not covered in binomial trees, even with many stochastic factors (in Chapter 22). Briefly, starting from the top of the table, the ex-coupon value at T = 2 is simply the principal 100. Then, we know that Duration of assets = x×Duration of leveraged inverse floater+(1−x)×Duration of T-bills (3.53) From x = 0.1366, the duration of leveraged inverse floaters (= 5.5040) and the duration of assets (= 2.89) Duration of T-bills = 2.89 − 0.1366 × 5.5040 = 2.4764 1 − 0.1366 (3.54) That is, the $20.5 billion Orange County portfolio could well have been mainly invested in short-term Treasury bonds (with duration of only 2.4764). Similarly, regulators may use the same tools to assess the fair valuation of complex securities, at least to first order, without needing a Ph.D. in mathematics or physics. They are quoted on annualized basis. Figure 1.7 reports the cumulative weekly failures of delivering Treasury securities, in millions of dollars, by primary dealers, and the 2008 spike is clearly visible. 2.1.2 Discount Factors over Time A second important characteristic of discount factors is that they are not constant over time, even while keeping constant the time-to-maturity T − t, that is, the interval of time between the two dates t and T in the discount factor Z(t, T). If a chapter is too simple, for instance, because it is only introductory, then it is hard to apply the concept to a real-world situation, which tends to be complicated. PART II: TERM STRUCTURE MODELS: TREES. Because expert arbitrageurs know this fact, some apparent mispricing may persist in the market place. Simon (Bentley College), Donald J. What happens if there is a hike in interest rates? Using the yield curve in Table 3.6, compute the dollar duration for the following securities: EXERCISES 101 (a) Long a 5-year coupon bond paying 4% semiannually (b) Short a 7-year zero coupon bond (c) Long a 3 1/2-year coupon bond paying 7% quarterly (d) Long a 2-year zero spread floating rate bond paid semiannually (e) Short a 2 1/4-year zero spread floating rate bond paid semiannually (f) Short a 5 1/4-year floating rate bond with a 25 basis point spread paid semiannually 5. The natural question is then the following: Which one of these measures is best? 7. For general information on our other products and services please contact our Customer Care Department with the U.S. at 877-762-2974, outside the U.S. at 317-572-3993 or fax 317-572-4002. Swaps market: A swap is a contract according to which two counterparties agree to exchange cash flows in the future. Using these two discounts Z y (0, T) instead of Z(0, T) in Equations 2.31 and 2.32, respectively, we indeed obtain the correct prices: Price T-note c=4.750 = 9.5 T =0.5 Price T-bond c=8.875 = 9.5 T =0.5 1+ 1+ 4.750/2 2×T + 3.7548% 2 8.875/2 2×T + 3.6603% 2 100 1+ 3.7548% 2 100 1+ 3.6603% 2 2×9.5 = 107.8906 2×9.5 = 141.5267 This example shows that there is something curious in the definition of yield to maturity. Why then do traders use the notion of yield to maturity y in their every day trading? The 3-year leveraged inverse floater has a duration of 5.5040, almost twice its maturity. Compute the Macaulay and modified duration for the same securities as in Exercise 1. The TIPS offer protection to investors against this possibility: Because the principal is adjusted for inflation, higher inflation translates into both a higher final payoff at maturity of the bond, and higher coupons as well, since the coupon is defined as a fixed percentage of current principal (which increases with inflation). The vertical axis represents the interest rate level r(t, t + m) that corresponds to the various maturities. The same investment strategy could be described in terms of a discount factor as well: The discount factor here is the INTEREST RATES 33 Figure 2.2 Expected Inflation 16 14 Expected Inflation Rate (%) 12 10 8 6 4 2 0 −2 1955 1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010 Data Source: Bureau of Labor Statistics. Maybe I should have not listened, but it is too late now. Examples are Treasury bills or STRIPS. It helps reduce the sensisitivy of equity to changes in interest rates, and ensures that cash flows received from assets are sufficient to pay the cash flows from liabilities. Hence, the trader delivers the bond as collateral to the repo dealer and receives the cash to purchase the bond. Chapter 9 begins the second part of the book, which concerns the fair valuation of derivative securities by no arbitrage. For each bond i = 1, .., N with coupon c i and cash flow payment dates maturity Tji , for j = 1, .., ni, the Nelson Siegel model implies that the bond price should be ⎛ ⎞ ni i c Z(0, Tji) + Z(0, T i)⎠ (2.54) Pci, NS model = 100 × ⎝ 2 j=1 For the same bond, we have the price quoted in the market, Pci, data (note that this has to be the invoice price and not the quoted price). The total value of the zeros the arbitrageur sells is $99.997 million, realizing approximately $2 million. At issue date t = 0, the price of a coupon bond with coupon rate equal to the constant semi-annual rate c = r 2 is equal to par. Historical Distribution Approach. For bond portfolios, the volatility is determined by movements in the interest rates. I also would like to thank Monika Piazzesi (Stanford University) and Jefferson Duarte (Rice University) for their early feedback, as well as Senay Agca (George Washington University), David T. At time T , the trader gets back the bond from the repo dealer, sells the bond in the market to get P T and pays (Pt − haircut) plus the repo interest to the dealer. We apply these continuous time methodologies and the rules of no arbitrage in Chapter 15 to compute the fair valuation of Treasury notes and bonds, as well as derivative securities, such as options. For instance, for given interest rate r n (t, T) with n compounding frequency, we can determine the continuously compounded interest rate r(t, T) by solving the equation e−r(t,T)(T −t) = Z(t, T) = 1+ 1 rn (t,T) n n×(T −t) (2.9) Because of its analytical convenience, in this text we mostly use the continuously compounded interest rate in the description of discount factors, and for other quantities. In particular, Panel B illustrates a decreasing term structure of interest rates, as occurred on November 30, 2000. Consider the case in which the spread s = 0. It changes the discount rate to apply to the future cash flows. Discuss whether this scenario is possible, and, if not, what arbitrage strategy could be set up to gain from the mispricing. Because of transaction costs and the lack of perfect co-movement among variables, some risks do in fact exist, and arbitrageurs must take them into account while they trade. The intuition is also quite straightforward: Inflation is exactly what determines the time value of money, as it determines how much goods money can buy. Data 16.8.5 Conclusion Appendix: Derivation of Delta for Call Options 563 565 570 570 570 572 572 575 575 576 578 579 581 584 585 586 588 590 RISK NEUTRAL PRICING AND MONTE CARLO SIMULATIONS 593 17.1 17.2 17.3 593 594 598 599 599 602 603 606 610 611 613 619 621 622 16.4 16.5 16.6 16.7 16.8 16.9 17 17.4 17.5 17.6 17.7 17.8 17.9 18 Risk Neutral Pricing Feynman-Kac Theorem Application of Risk Neutral Pricing: Monte Carlo Simulations 17.3.1 Simulating a Diffusion Process 17.3.2 Simulating the Payoff 17.3.3 Standard Errors Example: Pricing a Range Floater Hedging with Monte Carlo Simulations Convexity by Monte Carlo Simulations Summary Exercises Case Study: Procter & Gamble / Bankers Trust Leveraged Swap 17.9.1 Parameter Estimates 17.9.2 Pricing by Monte Carlo Simulations THE RISK AND RETURN OF INTEREST RATE SECURITIES 627 18.1 627 Expected Return and the Market Price Risk CONTENTS 18.2 18.3 18.4 18.5 18.6 18.7 19 631 631 633 635 636 639 642 644 648 648 649 NO ARBITRAGE MODELS AND STANDARD 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Why this Book? The 30-year coupon bond has a duration of 17.72, while the overnight deposit has zero duration, as the deposit rate resets daily. For instance, if the reference rate is the LIBOR, then the LIBOR curve should be used, as discussed in Chapter 5. Such options are implicitly embedded in numerous debt securities, from callable bonds to mortgage backed securities. If Ms. Caselli is not planning to consume out of her capital, this capital loss is of no consequence: She still possesses the same bond as before, which will keep paying the same $40,000 per year. The interesting fact about Figure 1.1, though, is the rise of another market, which has become a dominant market in the U.S., namely, the market of mortgage backed securities. The case study in Section 3.7 of Chapter 3 discusses the risk and return of such leveraged transactions. In this context, we also discuss the market for TIPS, the ROADMAP OF FUTURE CHAPTERS 27 inflation-protected debt securities. Finally, this chapter contains the academic evidence about the variation over time of interest rates, and the fact that risk premia to hold bonds are time varying. The slope of this tangent is the first derivative of P z (r, t, T) with respect to r, dPz /dr. In addition, the chapter introduces the notion of implied volatility, that is, the volatility of interest rates that is implied by the value of options. This fact is easily illustrated with an example: EXAMPLE 2.11 Columns 1 to 6 of Table 2.2 display coupon rates, maturities, and quotes of the latest issued Treasury notes on February 15, 2008. In this chapter we also introduce a popular pricing methodology called risk neutral pricing. This shape is called an inverted hump. 28 AN INTRODUCTION TO FIXED INCOME MARKETS 1.8 SUMMARY In this chapter we covered the following topics: 1. In particular, the dates corresponding to Panel C and Panel B of Figure 2.3 are only eight months apart, and yet the term structures are quite different in shape. In particular, we begin with simple, one-period binomial trees to explain the relations that have to exist between any pair of fixed income securities. Examples Each chapter contains many numerical examples illustrating the concepts introduced in the chapter. In particular, this chapter answers the question of why the term structure of interest rates, on average, slopes upward. Such coupon bond has a duration of 17.72. The U.S. Treasury does not issue floating rate bonds, but governments of other countries as well as individual corporations do. Real-world examples including the pricing of long-term structured derivatives illustrate how the methodology can be readily applied to price relatively complex securities. Investors in T-notes and T-bonds are subject to inflation risk: Because the payment of coupons and final principal is in nominal 2 The Treasury suspended the issuance of one year Treasury bills from August 2001 to June 2008. More specifically, in a reverse repo, the trader essentially (A) borrows the security from the dealer; (B) sell it in the market; and (C) post cash collateral with the dealer. That is, let r(t, T) be the continuously compounded term structure of interest rates at time t. An investor is planning a $100 million short-term investment and is going to choose among two different portfolios. Definition 3.2 The duration of a security with price P is the (negative of the) percent sensitivity of the price P to a small parallel shift in the level of interest rates. This increase in demand for U.S. Treasuries made it difficult or costly for traders who have short positions to find the bonds to return to their counterparties in the reverse repo transactions. Chapter 9 illustrates these important concepts in the simple framework of one-step binomial trees. Securities Industry and Financial Markets Association (SIFMA): This web site contains information and statistics about various fixed income markets, including Treasury Debt, Mortgage Backed Securities, Corporate Debt, Structured Finance, etc. 3.8 CASE ANALYSIS: THE EX-ANTE RISK IN ORANGE COUNTY'S PORTFOLIO In hindsight it seems that Orange County's investment strategy paved the way for its own disaster, but any reasonable assessment must be made using ex ante information. 8. The chapter applies the methodology to relatively complicated real-world securities, such as amortizing swaps and mortgage backed securities. Brown (University of Florida), Robert Jennings (University of Indiana), Robert Kieschnick (University of Texas at Dallas), David P. The seller of the bond is entitled to the portion of the coupon that accrued between the last coupon and the purchase date. Just like futures contracts, forward contracts allow institutions to lock in interest rates for the future. Term structure of interest rates: The term structure of interest rate is the relation between the interest rates and maturity. COUPON BONDS 2. As we have learned, this price defines a discount factor between the two dates of Z(t, T) = 0.95713. Indeed, the logic is the same as the one of the previous example: if two securities have same cash flows in the future but trade at different prices today, then an arbitrageur could buy the underpriced security and sell the overpriced one, realizing a profit today. Thus, the average time of coupon payments gets closer to today. One of the issues, though, is that bond data at six-month intervals are not always available. The next block shows the size of the mortgage backed securities and asset-backed securities markets. V. We can start from the left-hand side of the distribution, and move right until we count 1% of the observations. TIPS are issues with maturities of 5, 10 and 20 years. Figure 1.5 shows a schematic representation of the trade. 15 An investor has a long position in a bond if he holds the bond in the portfolio. We should mention that a spike in delivery fails can also be due to a snowball effect, as the failure to deliver from a security dealer implies that another security dealer who was counting on the delivery to THE REPO MARKET Table 1.6 Financing by U.S. Government Securities Dealers Reverse Repurchase and Repurchase Agreements (1) Average Daily Amount Outstanding 1981 - 2006 ($ Billions) Reverse Repurchase Repurchase 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 46.7 75.1 81.7 112.4 147.9 207.7 275 313.6 383.2 377.1 417 511.1 594.1 651.2 618.8 718.1 883 1,111.4 1,070.1 1,093.3 1,311.3 1,615.7 1,685.4 2,078.5 2,355.2 2,225.2 65.4 95.2 102.4 132.6 172.9 244.5 292 309.7 398.2 413.5 496.6 628.2 765.6 825.9 821.5 973.7 1,159.0 1,414.0 1,361.0 1,439.6 1,786.5 2,172.4 2,355.7 2,868.2 3,288.4 3,388.3 (1) Figures cover financing involving U.S. government, federal agency, and federal agency MBS securities. Because most Treasury securities have similar THE REPO MARKET 17 characteristics in terms of liquidity, market participants require the same interest rate for collateralized borrowing. The duration of the annuity promised to Ms. Caselli is about 12.35. Second, the variation of discount factors over time is rather substantial. Combining Equations 3.1 and 3.21 we obtain: Fact 3.4 For a nonzero valued security or portfolio with price P , the relation between duration and dollar duration is DP$ = P × DP (3.22) 85 DURATION In this case, the relation between the dollar duration of the portfolio and the dollar duration of its individual components is given by the sum of the dollar durations: $ is given by Fact 3.5 The dollar duration of portfolio of n securities, denoted by D W $ DW = n Ni Di$ (3.23) i=1 where Ni is the number of units of security i in the portfolio, and D i$ is the dollar duration of security i. Differential equations are introduced only through examples, as my aim here is to provide students with the notion of differential equations, and not the xxvi PREFACE methodology to solve for them. Because it is always the case that market participants prefer $1 today to $1 in the future, the following is true: Fact 2.1 At any given time t, the discount factor is lower, the longer the maturity T . What keeps these markets together? For longer maturities, the Treasury issues securities that carry a coupon, that is, they also pay a sequence of cash flows (the coupons) between issue date and maturity, in addition to the final principal. Indeed, the three agencies Fannie Mae, Freddie Mac, and Ginnie Mae hold or guarantee about half of the roughly $9 trillion U.S. mortgage market. Anyone who directly or indirectly either invest in fixed income securities or borrow money is subject to interest rate risks. Today is May 15, 2000, and the current, semi-annually compounded yield curve is in Table 3.6. Compute the duration for the following securities: (a) 3-year zero coupon bond (b) 3 1/4-year coupon bond paying 6% semiannually (c) 1-year coupon bond paying 4% quarterly 100 BASICS OF INTEREST RATE RISK MANAGEMENT Table 3.6 Yield Curve on May 15, 2000 Maturity Yield Maturity Yield Maturity Yield 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00 2.25 2.50 6.33% 6.49% 6.62% 6.71% 6.79% 6.84% 6.87% 6.88% 6.89% 6.88% 2.75 3.00 3.25 3.50 3.75 4.00 4.25 4.50 4.75 5.00 6.86% 6.83% 6.80% 6.76% 6.72% 6.67% 6.62% 6.57% 6.51% 6.45% 5.25 5.50 5.75 6.00 6.25 6.50 6.75 7.00 7.25 7.50 6.39% 6.31% 6.24% 6.15% 6.05% 5.94% 5.81% 5.67% 5.50% 5.31% Notes: Yields are calculated based on data from CRSP (Daily Treasuries). Wiley also publishes its books in a variety of electronic formats. On the other hand, deposits have a duration close to zero, as the short term interest rate needs to be adjusted frequently as market conditions change. Consider once again the U.S. market: As of the end of 2008, the U.S. debt stood at about $6 trillion, approximately 90% of which is in Treasury securities that indeed have a fixed income, namely, with constant coupons that are paid over time. In particular, denoting c i the coupon rate of bond i, we have ci (Tj) = 100 × ci /2 for Tj < T i and ci (T i) = 100 × (1 + ci /2) and finally ci (Tj) = 0 for Tj > T i . In this case we obtain a 99% 6-month VaR equal to $1.48 billion, close to the actual loss suffered by Orange County. While this part is self contained, as it contains all of the important mathematical concepts, readers should be ready to see a substantial step up in the analytical requirement compared to the previous two parts of the book, which, as mentioned, instead only require a background in basic calculus. That is, y is defined by the equation Pc (t, T) = n i=1 c/2 × 100 (1 + y/2) 2×(Ti −t) + 100 2×(Tn −t) (1 + y/2) (2.30) Before moving to interpret this measure of return on investment, it is important to recognize a major distinction between the formula in Equation 2.30 and the one that we

obtained earlier in terms of discount factors, namely Equation 2.15. The CCT semi-annual coupon is equal to the most recent rate on the six-month BOT (the Italian Treasury bill), plus a spread (fixed at 0.15%). (a) What is the value of the unhedged portfolio? The answer is no arbitrage, that is, the possibility does not exist for arbitrageurs to take large positions in different securities whenever the prices across markets do not line up. Therefore, the loan contract is equivalent to a standard floating rate loan contract plus an option that pays if interest rates become too high, just like in the example of the corporation above. 18 AN INTRODUCTION TO FIXED INCOME MARKETS Figure 1.5 Reverse Repo Transaction time t MARKET sell bond at P t == == get Pt TRADER buy bond at P T == == pay P T give bond back ==⇒ TRADER REPO DEALER ⇐= n get back P t ×(1+ repo rate × 360) borrow bond ⇐= REPO DEALER ==⇒ use P t as cash collateral time T = t + n days MARKET repo transactions to implement a relative value arbitrage trade on the yield curve through a dynamic long/short strategy. This can be seen by reorganizing Equation 3.1 as follows: Fact 3.1 Given a duration D P of a security with price P , a uniform change in the level of interest rates brings about a change in value of Change in portfolio value = dP = −D P × P × dr (3.2) EXAMPLE 3.1 A $100 million bond portfolio has duration equal 10, D P = 10. A withdrawal of funds is the worst nightmare for a bank, as depositors' money is not in the bank any longer: It has been loaned to others. An arbitrageur could purchase, say, $98 million of the 2-year note, and sell $2.1875 million of the 6-month, 1-year and 1.5-year zero coupons, and $102.1875 million of the 2-year zero coupon bond. The duration analysis in the previous sections can be applied more generally to analyze the relative potential duration mismatch between assets and liabilities. A homeowner who financed the purchase of his or her home using a adjustable rate mortgage (ARM), for instance, most likely also bought (probably unknowingly) an option against an increase in interest rates. Acknowledgments. (b) What is the total value of the portfolio now? The link between risk and return of fixed income securities is the object of Chapter 18. This implies that one basis point increase in the level of interest rates dr = .01% generates a swing in portfolio value of Change in portfolio value = dP = −10 × $100 million × .01/100 = −$100, 000 That is, the portfolio manager stands to lose $100,000 for every basis point increase in the term structure. There is a reason why the final wealth came up positive – due to the convexity of bond prices with respect to interest rates – that we discuss in Chapter 4. Invest all of $1,000,000 in 6-months T-bills. The most interesting feature of "muni" bonds is that the interest income from their coupons are tax-exempt. EXAMPLE 3.2 A portfolio manager has $100 million invested in 5-year STRIPS. The VaR formula used in Equation 3.31 includes the expected change in the portfolio μP = −DP ×P ×E[dr]. The computation on the expected change E[dr] is typically very imprecise, and standard errors are large. The chapter provides numerous real-world examples, as well as a case study discussing the fair valuation of the leveraged swap between Bankers Trust and Procter & Gamble, which was a the center of a famous court case in 1994. This VaR measure implies that there is only 5% probability that the portfolio losses will be higher than $3 million over the next month. This chapter discusses the basics of interest rate risk management. So, for the coupon payments, this is the same as having a long position in a fixed rate bond and a short position in a floating rate bond, as such positions would entail receiving a fixed coupon and paying a floating coupon. Let the two coupon rates be 15% and 3%. How can we compute the duration of a security? The figure shows that indeed for APPENDIX: EXTRACTING THE DISCOUNT FACTORS Z(0, T) FROM COUPON BONDS 69 most bonds this is the case, indicating that the model is doing quite well. Similarly, given a discount factor Z(t, T), we can obtain the semiannually compounded interest rate. In fact, $95.713 × (1 + 4.43%/2)2 = $100. For instance, if c c c × Pz (t, Tn) (2.22) Pc(t, Tn) < × Pz (t, T1) + × Pz (t, T2) + ... Equation 3.16 shows that the duration of a floating rate bond is simply equal to the time left to the next coupon payment date Ti+1 − t. Indeed, pure arbitrage strategies are rare in the market. (a) Compute the prices and the yields to maturity of these coupon bonds. To simplify our analysis, we will only consider continuously compounded interest rates, as in the definition above. × (1 + 4.89%/12) = $105. CASE STUDY: ORANGE COUNTY INVERSE FLOATERS Table 2.6 Bond Quotes on December 31, 1993 Maturity Coupon Bid Ask 19940106 19940113 19940115 19940120 19940127 19940131 19940203 19940210 19940215 19940215 19940215 19940217 19940224 19940228 19940303 19940310 19940317 19940324 19940331 19940331 19940331 19940407 19940414 19940415 19940421 19940428 19940430 19940505 19940512 19940515 19940515 19940515 19940519 19940526 19940531 19940602 19940609 19940616 19940623 19940630 19940630 19940630 19940715 19940728 19940731 19940815 19940815 19940815 19940815 19940825 19940831 19940922 19940930 19940930 19941015 19941020 19941031 19941115 19941115 19941115 19941115 19941117 19941130 19941215 19941231 19941231 19950115 19950131 19950215 19950215 19950215 19950215 19950215 19950215 19950228 19950331 0.000 0.000 7.000 0.000 0.000 4.875 0.000 0.000 9.000 6.875 8.875 0.000 0.000 5.375 0.000 0.000 0.000 0.000 5.750 8.500 0.000 0.000 0.000 7.000 0.000 0.000 5.375 0.000 0.000 7.000 9.500 13.125 0.000 0.000 5.125 0.000 0.000 0.000 0.000 5.000 8.500 0.000 8.000 0.000 4.250 8.750 6.875 8.625 12.625 0.000 4.250 0.000 4.000 8.500 9.500 0.000 4.250 10.125 6.000 8.250 11.625 0.000 4.625 0.000 4.625 7.625 8.625 4.250 3.000 10.500 5.500 7.750 11.250 3.875 3.875 99.960 99.904 100.094 99.841 99.788 100.125 99.724 99.666 100.688 100.406 100.656 99.612 99.551 100.344 99.489 99.423 99.367 99.306 100.625 101.250 99.248 99.173 99.113 101.031 99.047 98.990 100.656 98.920 98.849 101.313 102.219 103.563 98.780 98.714 100.719 98.653 98.582 98.516 98.439 100.813 102.500 98.391 102.406 98.113 100.469 103.188 102.063 103.125 105.531 97.841 100.469 97.563 100.313 103.594 104.438 97.249 100.531 105.438 101.969 103.875 106.750 96.924 100.875 96.646 100.906 103.813 104.875 100.531 100.250 107.344 101.844 104.281 108.125 100.063 100.031 99.961 99.908 100.156 99.846 99.795 100.188 99.728 99.671 100.750 100.469 100.719 99.617 99.557 100.406 99.492 99.427 99.371 99.311 100.688 101.313 99.253 99.178 99.119 101.094 99.053 98.997 100.719 98.927 98.856 101.375 102.281 103.625 98.788 98.723 100.781 98.661 98.591 98.525 98.449 100.875 102.563 98.401 102.469 98.125 100.531 103.250 102.125 103.188 105.594 97.854 100.531 97.578 100.375 103.656 104.500 97.265 100.594 105.500 102.031 103.938 106.813 96.942 100.938 96.665 100.969 103.875 104.938 100.594 101.250 107.406 101.906 104.344 108.188 100.125 100.094 Maturity 19950415 19950430 19950515 19950515 19950515 19950515 19950531 19950630 19950715 19950731 19950815 19950815 19950831 19950930 19951015 19951031 19951115 19951115 19951115 19951130 19951231 19960115 19960131 19960215 19960215 19960229 19960331 19960415 19960430 19960515 19960515 19960531 19960630 19960715 19960731 19960815 19960831 19960930 19961015 19961031 19961115 19961115 19961130 19961231 19970115 19970131 19970228 19970331 19970415 19970430 19970515 19970531 19970630 19970715 19970731 19970815 19970831 19970930 19971015 19971031 19971115 19971130 19971231 19980115 19980131 19980215 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103.219 112.625 105.063 116.125 100.844 118.375 118.000 116.844 136.219 113.719 145.313 114.344 147.875 113.813 164.031 111.563 155.656 111.844 104.250 140.344 134.813 103.250 135.313 138.656 99.625 144.750 149.969 161.406 145.219 149.375 139.531 128.906 154.250 147.375 138.750 131.531 108.188 111.031 125.938 127.594 130.969 129.625 128.219 119.094 123.906 127.219 127.281 116.438 119.719 119.719 118.406 109.094 114.156 108.156 98.656 101.188 100.125 112.563 100.531 116.813 98.438 98.438 108.250 98.250 99.969 115.813 99.688 99.688 105.094 116.219 107.906 117.938 105.063 113.000 103.281 112.688 105.125 116.188 100.906 118.438 118.063 116.906 136.344 113.781 145.438 114.406 148.000 113.875 164.156 111.625 155.781 111.906 104.313 140.469 134.938 103.313 135.438 138.781 99.688 144.875 150.094 161.531 145.344 149.500 139.656 129.031 154.313 147.438 138.813 131.594 108.250 111.094 126.000 127.656 131.031 129.688 128.281 119.156 123.969 127.281 127.344 116.500 119.781 119.781 118.469 109.156 114.219 108.219 98.719 Data excerpted from CRSP (Daily Treasuries) ©2009 Center for Research in Security Prices (CRSP), The University of Chicago Booth School of Business. The chapter offers several applications, and further highlights the pros and cons of different types of models. The interest rate swaps market, which was negligible at the beginning of the 1980s, experienced an exponential growth, reaching $328 trillion (notional) by December 2008. ERisk Case, Orange County (2001), page 2. This exercise, however, may prove more challenging than it sounds. 8 Data excerpted from CRSP (Daily Treasuries) ©2009 Center for Research in Security Prices (CRSP), The University of Chicago Booth School of Business. The general formula for a semi-annual floating rate bond with zero spread s is PF R (t, T) = Z(t, Ti+1) × 100 × [1 + r2 (Ti)/2] (2.40) where Z(t, Ti+1) is the discount factor from t to T i+1 . Typically, we think of a derivative security as a security whose value can be derived from the value of another, more primitive security from the rules of no arbitrage. Finally, the loss was dP = −1.6. Substituting into Equation 3.45 we find 1 −1.6 = 7.90 (3.46) Duration = − 7.5 .027 That is, if Orange County's portfolio was invested only in Treasury securities, given the ex-post losses, we would gather that the duration of his portfolio should have been around 7.90. From its value of only $372 billion in 1985 it increased steadily over time, to become larger than the U.S. debt market in 1999, and to become $3 trillion larger than the U.S. debt market by December 2008. The third part of the book is mathematically more advanced, and some familiarity with advanced calculus is required. Similarly, forward rate agreements and futures contracts also grew over time, although at a much slower rate. The VaR measures the maximum loss with 95% probability. I always use data to set up an example or to illustrate a concept, not only because it makes the lesson more relevant, but because it shows that we can actually tackle real-world valuation problems by studying the concepts illustrated in each chapter. If we vary the maturity T , making it longer or shorter, the discount factor varies as well. The higher the expected inflation, the less appealing it is to receive money in the future compared to today, as this money will be able to buy a lesser amount of goods. Asset-Liability Management: This is a strategy of choosing the (dollar) duration of liabilities to match the (dollar) duration of assets. While it sounds intuitive that we should always use the most conservative measure of risk, i.e. the one implying the largest possible losses, there are good reasons also to not be too conservative, as a portfolio manager who is overly conservative may miss important profit opportunities. THE MORTGAGE BACKED SECURITIES MARKET AND ASSET-BACKED SECURITIES MARKET 21 Figure 1.7 Primary Dealers Fails to Deliver: 1990 to 2009 6 3 x 10 Fails (in Millions of Dollars) 2.5 2 1.5 1 0.5 0 07/04/1990 07/07/1993 07/03/1996 07/07/1999 07/03/2002 07/06/2005 07/02/2008 Source: Federal Reserve Bank of New York. There is a six-month temporal lag between the determination of the coupon and its actual payment. Understanding the forces that move the term structure of interest rates is important to determining what will happen to these prices once the crisis is over. 3.8.2 Conclusion The ex-ante measurement of risk is difficult and full of potential pitfalls. Bank for International Settlements (BIS): This web site contains research on financial markets and institutions, the Basel committee on banking supervision, statistics about OTC derivative markets , and much more. For instance, the 4.375% Treasury note issued on January 3, 2006 and with maturity of December 31, 2007, pays a cash flow of $2.1875 on June 30, 2006, December 31, 2006, and June 30, 2007, while it pays $102.1875 on December 31, 2007. 3.7.3 The Risk in Inverse Floaters Although the main reason for having a record-breaking loss in the Orange County portfolio was leverage, the trigger was the change in interest rates and its effect on inverse floaters. Yet, the large leverage and the very high duration of leveraged inverse floaters may still have produced large losses as the interest rate increased. In this case, some approximation is necessary. In well-functioning markets, there shouldn't be (large) arbitrage profits that are left on the table, as arbitrageurs would step in and trade them away. Thus, a trader can quote the same bond by using P c or y. More specifically, Equation 3.1 defines duration as 1 dP Duration = DP = − (3.45) P dr We know that the portfolio value before the hike in interest rates was approximately $7.5 billion, and thus P = 7.5. In addition, the change in the level of interest rates was dr = 6.7% − 4% = 0.027. In this case, the definition of the modified duration as the (negative of the) sensitivity of prices to changes in interest rates (Equation 3.17) does not correspond exactly to the simple formulas derived earlier, and a small adjustment is needed. In addition, some part of the gain would also accrue to the investment bank that brokers the deal. Was this luck? Would the difference in yields imply that one is a better "buy" than the other? A solutions manual is available to instructors. After some careful (and time consuming) analysis of the data in Table 2.6, we resolve to use the subsample of data, contained in Table 2.7, which are nicely spaced at 6-month intervals. The prerequisites for my course include an investment course and a basic option course, although I often let students with a solid mathematical background to take the course without the prerequisites. Many financial institutions have a duration mismatch between their assets and their liabilities. First, if the spread s is nonzero we can decompose the total cash flow per period in two parts, the floating part and the fixed part. In particular, recall from Chapter 2, Equation 2.43, that we can write the price of a (plain vanilla) inverse floater with maturity T with annual payments and coupon c(t) = c − r1 (t − 1) as Price inverse floater PIF (0, T) = Pz (0, T) + Pc (0, T) − PF R (0, T) (3.49) 106 BASICS OF INTEREST RATE RISK MANAGEMENT Table 3.8 The Duration of the 15% Fixed Rate Bond Date Cash Flow Discounted Cash Flow Weight w T w∗T 12/31/1994 12/31/1995 12/31/1996 0.15 0.15 1.15 0.1446 0.1379 1.0057 0.1123 0.1070 0.7807 1 2 3 0.1123 0.2141 2.3421 Total Value 1.2884 Duration: 2.6685 where Pz (0, T) is the price of a zero coupon bond, P c(0, T) is the price of a c coupon bond, and PF R (0, T) is the price of a floating rate bond, all of them with maturity T . So, he can compute what the value is at time t = 0.5. Suppose that at time t = 0.5 the interest rate is r 2 (0.5) = 3%, then the coupon at time T = 1 is c(1) = 100 × r2 (0.5)/2 = 1.5. This implies that the value of the bond at time t = 0.5 is equal to Value bond at 0.5 = Present value of (100 + c(1)) = 100 + 1.5 = 100, 1 + 0.03/2 which is a round number, equal to par. For instance, focusing on the last part of the period, 2000 - 2006, we see that while the 3-month rate moved from over 6% down to 1%, then went back up to 5%, the 5-year rate moved from 6% down to 3%, and then back up to 5%. In this chapter I tackle the issue of how to estimate the model's parameters, and show the potential shortcomings of the model. The following example illustrates the reasoning: EXAMPLE 2.9 On t = June 30, 2005, the 6-month Treasury bill, expiring on T 1 = December 29, 2005, was trading at $98.3607. For now, we only consider Treasury notes and bonds. Fortunately, the generalization to the more realistic case is simple. Is there an arbitrage opportunity? × 1 + = $105.11, 12 12 34 BASICS OF FIXED INCOME SECURITIES which is higher still. The 99% VaR is computed as in Equation 3.31 except that the number "1.645" is substituted by "2.326." This result of course relies on Equation 3.28, which is only an approximation. 6. Moreover, in some chapters, the exercise requires students to carry out Monte Carlo simulations, on the binomial trees, to value real-world fixed income securities with embedded options, such as the Bermudan callable bonds of Freddie Mac. Treasury dealers quoted this price as d= 100 − 97.477 360 × = 4.99% 100 182 where d is an annualized discount rate on the face value of the Treasury bill. settle his or her own obligation may be unable to deliver as well, and so on, generating a domino effect.4 Starting May 1, 2009, the Federal Reserve imposed a penalty charge of 3% over its Fed funds rate for failing to deliver the bonds in the repo transactions the Federal Reserve conducts daily in its open market operations (see Chapter 7 for details on the Fed conduct of monetary policy). c1 (T2) c2 (T2) ... For convenience, assume that annuity payments and the long-term bond coupon payments occur on the same date. The (100−α)% T Value-at-Risk of a portfolio P is the maximum loss the portfolio can suffer over the horizon T with α% probability. The figure also shows that indeed at T = 2.5 there is a star that differs substantially from the diamond. The estimation proceeds as follows. In essence, the idea is to assume that the discount function Z (t, T) is given by a weighted average of basis functions f (T), where the weights are chosen to best match the bond prices. Additionally, through Equations 2.26 and 2.27 we can convert coupon paying bonds into zero coupon bonds. 1.3.2 Eurodollar Rate The Eurodollar rate is the rate of interest on a dollar deposit in a European-based bank. Although expected inflation is the most obvious culprit in explaining the variation over time of discount factors, it is not the only one. I use case studies also to show that we must often make many approximations when we apply relatively simple formulas or models to real-world data. Thus, quite immediately, we have Return on zero coupon bond = 1 −1 Z(t, T) (2.28) This is the return between t and T . In percentage terms, the trader only put up the haircut (the margin) as own capital. (2.33) yc=8.875 = 3.6603% (2.34) As it can be seen from Equations 2.33 and 2.34, the bond with the higher coupon has lower yield to maturity y. Let's put some numbers down to see this more precisely. Chapter 10 extends the analysis to multistep trees. How can we compute a 6-month VaR? The decision to rely foremost on the use of data as a pedagogical device springs from my beliefs that only by doing the analysis with real-world numbers can a student really understand not only the concepts illustrated in the particular chapter, but also the complexity of applying models to the real world. Translating such number into another compounding frequency is immediate from Equation 2.9, which, more explicitly, implies rn (t, T) (2.10) r(t, T) = n × ln 1 + n r(t,T) (2.11) rn (t, T) = n × e n − 1 To conclude, then, this section shows that the time value of money can be expressed equivalently through a discount factor, or in terms of an interest rate with its appropriate compounding frequency. Chapter 6 is the second introductory chapter on derivative securities, covering futures and options. A floating rate coupon bond is like a standard coupon bond, but its coupon is indexed to some other short-term interest rate, which changes over time. 5 Interest Rate Derivatives: Forwards and Swaps. Let dP denote the P&L of the portfolio, so that the loss is LT = −dP . For each given set of parameters (θ 0 , θ1 , θ2 , λ) we can compute the difference between model prices and data. Then, we know that at the next reset date, time 0.5, the ex-coupon value of the floating rate bond will be $100. Is there any one that seems to be mispriced? Typically, in the U.S. the term spread is positive. 2 2 c × 100 ... The value of what $1 in the future would be in today's money is called the discount factor. In particular, we discuss the fact that if a security is providing an above-market coupon or rate of return, then most likely this security is exposed to some risk, which perhaps is not made completely explicit. This example illustrates how the type of interest rate risk management that an institution or a person may want to engage in depends on the goals of the institution or individual. General Collateral Rate (GCR): This is the repo rate on most Treasury securities, such as the off-the-run Treasuries. Chapter 17 introduces the notion of risk neutral pricing in continuous time models. Using the discount factors Z(0, T) in Table 2.2 and the formula in Equation 2.15 we can determine the fair prices of the two securities. Or are there instances in which the final wealth was negative? DURATION 75 3.1.1 The Savings and Loan Debacle The savings and loan debacle in the 1980s is a standard example of what can go wrong when interest rates shift. (b) What is PV01 for each portfolio? 2.8.2 Calculating the Term Structure of Interest Rates from Coupon Bonds The next challenge is to determine the term structure of the interest rates so as to obtain the discount rates for the bonds. 2.5.2 Complications We must discuss two simplifying assumptions made above: First, the spread s on the floating rate is zero. It is instructive to go through the steps to compute the duration of a zero coupon bond, D z,T , where the notation "z" reminds us that this calculation is done for a zero coupon bond. At the basis of much of the analysis is the law of one price, discussed next: Fact 1.1 The law of one price establishes that securities with identical payoffs should have the same price. Should she invest in long-term bonds or short-term bonds? The methodology is just a slight modification of the VaR computation. Indeed, we can consider the total assets of the firm as a portfolio of securities (e.g. individual loans, receivables, and so on) and thus use the earlier formula in Equation 3.8 to compute the duration of assets as a weighted average of the durations of its components. Pc(0, Tn) = n−1 c/2 × 100 2×Ti i=1 (1 + r2 /2) + 100 × (1 + c/2) 2×Tn (1 + r2 /2) = 100 (2.16) To understand the above fact, consider a 1-year note. I also thank Jennifer Manias, the Associate Editor, for helping with the logistics of the publication process. Forward contracts are similar to futures contracts, in that two counterparties agree today that they will exchange a security (or cash) in the future at a price that is decided today. Indeed, the large expansionary monetary policy of the Federal Reserve, which was necessary to keep the banking sector from collapsing due to lack of liquidity, may spur a bout of inflation in the future. To provide an example, Figure 1.3 plots the life cycle of a 20-year bond, from its issuance in February 1986 to its maturity on February 2006.1 The variation over time of the price of the bond is quite stunning, with run ups of over 30 percent within two years (e.g., between 1991 and 1993), and run downs at even higher speed (e.g., 1994). Consider the data in Table 2.4. Consider two bonds, both with 7 years to maturity, but with different coupon rates. This is evident in the decrease in the face value of government debt during this period. The concept that I use throughout is the one of no arbitrage and the law of one price, that is, the fact that two securities that have the same cash flows should have the same price. In Part III, the exercises again rely on real-world data to fit more complex models of the term structure, and ask students to price relatively complex securities. Compared to the second part of the book, which also accomplishes similar goals, the concepts discussed in this part of the text are more realistic, and moreover provide analytical formulas for the pricing and hedging of numerous securities, a very convenient property for traders who are pressed for time. In this book, we cover several models, and we go over their properties, the approximations of reality they make, and their possible drawbacks. Investing in Bonds: Market at a Glance This web site is dedicated to Fixed Income investors. The chapter also introduces the Federal funds futures, and the information contained in such derivative contracts to predict future movements in the Federal funds rate. Therefore, μP = −5 × 100 × μ = −.0033 and σ P = 5 ×100 ×σ = 2.0767. We can put these cash flows in a row vector as follows: Ci = ci (T1) , ci (T2) , ..., ci (Tn) We can denote by Z (0) the vector of discount factors for various maturities T i , that is ⎛ ⎜ ⎜ Z (0) = ⎜ ⎝ Z (0, T1) Z (0, T2) .. The initial spur to swap trading was due to exploit arbitrage opportunities. The figure however also shows that sustained periods of delivery fails occurred in the past as well, such as in 2001 and in 2003. The reasoning is that every day, quotes are available on bonds maturing at different dates. 30/360: Assume there are 30 days in a month and 360 in a year; 3. Table 1.4 provides a snapshop of the issuing activity of the U.S. Treasury as of July 15, 2009. The concept of negative convexity is thoroughly discussed, and illustrated by using data from the main trading market of agency pass throughs, the To-Be-Announced (TBA) market. 86 BASICS OF INTEREST RATE RISK MANAGEMENT The dollar losses due to a basis point increase in the level of interest rates, as computed in Equation 3.25 is a common measure of interest rate risk. 77 DURATION In this case the constant A is the notional 100, the constant a equals the time to maturity T − t, and the variable x equals the continuously compounded interest rate r. Overall, we observe that Firm A pays 14% instead of 15%, which was its fixed coupon market rate. A large recent advance in academic literature links no arbitrage models with the activities of central banks, and this is important. While the U.S. government does not issue floating rate bonds, other governments do. The compounding frequency is a crucial element that must be attached to the interest rate figure. For instance, for normally distributed variables, we have the following result: Fact 3.7 Consider Fact 3.6. Under these assumptions: f(−1.645) 95% Expected shortfall = − μP − σ P × N (−1.645) = − (μP − σ P × 2.0628) (3.33) (3.34) 90 BASICS OF INTEREST RATE RISK MANAGEMENT where f(x) denotes the standard normal density and N (x) is the standard normal cumulative density,5 The 99% expected shortfall is obtained as in Equation 3.34 except with the number "2.6649" in place of "2.0628." A quick comparison of Equations 3.34 and 3.31 shows that for the normal distribution case, the expected shortfall contains the same information as the Value-at-Risk, as the only difference is the coefficient that multiplies σ P . Duration: The (negative of the) percentage sensitivity of a security to parallel shift in the term structure of interest rates is known as duration. This investor is seriously worried about interest rate volatility in the market. The term T of the repo transaction is decided at initiation, i.e., time t. 3.3 INTEREST RATE RISK MANAGEMENT Interest rate risk management is a key activity for banks, bond portfolio managers, corporations, governments, and, in fact, households. To price this security, we need to revise the steps we took to price inverse floaters. Finally, Panel D plots a term 4 We calculated the spot curves using the extended Nelson Siegel model (see Section 2.9.3 in the appendix at the end of the chapter) based on data from CRSP (Monthly Treasuries) ©2009 Center for Research in Security Prices (CRSP), The University of Chicago Booth School of Business. The entries in Table 1.2 summarizes these channels. Every coupon bond i is characterized by a series of cash flows and a maturity T i . This methodology involves using computers to simulate interest rate paths and price paths and then using those simulated quantities to compute current prices and hedge ratios. Several applications show the usefulness of these new models to obtaining the price of even more complicated securities, although often by relying on Monte Carlo simulations. First, the duration of the levered portfolio now has to be computed using the assets and liability formula in Equation 3.44, that is (3.47) Duration leveraged portfolio = w A × DA + wL × DL where wA = A/(A − L) = 20.5/7.5 and wL = −L/(A − L). • If the interest rate r 2 (0.5) declines, the future cash flow declines. For instance, the U.S. Treasury issued a 182-day bill on t1 = August 26, 2004, with maturity T 2 = February 24, 2005, for a price of $99.115. Figure 2.5 The Term Structure over Time 0.18 3 months 1 year 5 years 0.16 0.14 Interest Rate (%) 0.12 0.1 0.08 0.06 0.04 0.02 0 1955 1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010 Source: Center for Research in Security Prices Why does the term spread change over time? Is such an investment riskier than an investment in Treasury securities? If cash flow comes from the amount of capital available, the sensitivity of capital itself to interest rates becomes a big issue. Between coupon dates, interest accrues on the bond. Finally, we note that dealing with real data and real markets often poses additional problems in the valuation and risk analysis of fixed income instruments: For instance, the computation of the discount curve Z(0, t) requires the analysis of the data in Table 2.6, which is not straightforward. The mortgage backed securities trade in the market. Of course, these types of pure arbitrage opportunities are hard to find in financial markets. Treasury notes (T-notes) are medium-long term debt instruments, with maturity up to 10 years. 3.2.4 Duration and Average Time of Cash Flow Payments While we have derived the formula for duration in Equation 3.13 from the definition of duration as the percentage sensitivity of a security to changes in interest rates (see Definition 3.2), some confusion sometimes arise about the notion of duration because sometimes people define duration as the average time of payments, as in Equation 3.13 (see also Section 3.2.6). There are two firms, Firm A and Firm B. We begin with government debt, appearing under the heading of U.S. T-Bills and U.S. Bonds in Table 1.2. 1.2 THE GOVERNMENT DEBT MARKETS Essentially all countries issue debt to finance their operations. Figure 2.6 The Term Structure of Interest Rates on December 31, 1993 5.5 5 Yield (%) 4.5 4 3.5 Bootstrap Nelson Siegel Extended Nelson Siegel 3 0.5 1 1.5 2 2.5 3 Time to Maturity 3.5 4 4.5 5 Source: Center for Research in Security Prices. For this reason, it is often more accurate to consider only the unexpected VaR, that is, consider only the 95% loss compared to the expected P&L μP . The set of parameters (θ0 , θ1 , θ2 , λ) that achieves this objective would be the one to use to determine the discount factors Z(0, T). They are thus an example of zero coupon bonds, bonds with no intermediate cash flows between issue date and maturity. Unfortunately, this procedure requires all of the bonds, because otherwise the iterative procedure stops and there is no way to keep going. Conclusion ix Interest Rate Futures 6.1.1 Standardization 6.1.2 Margins and Mark-to-Market 6.1.3 The Convergence Property of Futures Prices 6.1.4 Futures versus Forwards 6.1.5 Hedging with Futures or Forwards? The main money market rates are the commercial paper rate, LIBOR, and the Federal funds rate. I also thank John Heaton (The University of Chicago Booth School of Business), Jakub Jurek (Princeton University), Nick Roussanov (The Wharton School, University of Pennsylvania), and Richard Stanton (The Hass School of Business, University of California at Berkeley) for being so brave to adopt an early draft of this book in their MBA or Master courses, so that I could collect very valuable feedback from them and their students. On this date, the 6-month, 1-year, 1.5-years, and 2-year discounts were Z(t, t + 0.5) = 0.97862, Z(t, t + 1) = 0.95718, Z(t, t + 1.5) = 0.936826 and Z(t, t + 2) = 0.91707. However, as shown in Table 1.2, longer-term agreements reach 30 days or even more. (b) What is the new dollar duration of each portfolio? In particular, most repurchase agreements are for a very short term, mainly overnight. That is, the spread was zero both at the beginning and at the end of this sample, but it was large in the middle. You are worried about interest rate volatility. 10 AN INTRODUCTION TO FIXED INCOME MARKETS Figure 1.3 The February 2006, 9.375%, 20-Year Bond Price Path 135 130 125 Price 120 115 110 105 100 95 1986 1988 1990 1992 1994 1996 1998 2000 2002 2004 2006 Source: Center for Research in Security Prices the bond and its maturity (e.g., 30 years) the U.S. enters a period of sustained inflation, the effective value of coupons and principal decreases, as investors cannot purchase as many consumption goods. Students find it very useful to see the same concepts introduced in binomial trees repeated in a continuous time framework, as their intuition gets solidified, especially through the plentiful examples. For interest rate futures, such as Eurodollar or Fed funds, the quoted "price" is given by "100− futures rate." For instance, the 90-day Eurodollar futuresEurodollar,futures contract with maturity "December," quoted at 95.07 in Table 1.2, establishes the rate today, 4.93% = 100 − 95.07, at which the party long the futures could deposit dollars in the Eurodollar market in December for the following 90 days.6 The futures market thus provides a convenient way for market participants to lock-in a future interest rate: For instance, a corporation that has a large receivable due in December can exploit the futures market to lock in the rate (4.93%) at which it can park the sum of money for the following 90 days. On February 15, 2008, traders could buy or sell two Treasury securities with the same maturity T = 9.5 years, but with very different coupon rates. In contrast, as we shall see below, the related notion of an interest rate is not un-ambiguous, as it depends on compounding frequency, for instance. The maturity of the latter Treasury note is T2 = June 30, 2006. Why does the coupon rate affect the yield to maturity? In fact, the top panel of Figure 3.7 shows that the volatility of the level of interest rates had been relatively low in the decade before 1994. The cash flow channel: If interest rates go up, the actual cash flow is reduced because coupon payments move inversely to interest rates. The problem with bootstrap is that correcting for these sources of imprecision is hard. The strategy is risk free, because at each coupon date in the future, the arbitrageur receives $2.1875 million from the Treasury, which he simply turns over to the investors who bought the zero coupon bonds. ⎞ ⎟ ⎟ ⎟ ⎠ Z (0, Tn) The price of a coupon bond can be written using vector multiplication as: Pci (0, T) = Ci ×Z (0) We can denote the column vector of bond prices available at time 0 as ⎛ ⎜ ⎜ P (0) = ⎜ ⎝ Pc (0, T1) Pc (0, T2) .. The procedure is otherwise the same as in the case of the Nelson Siegel model. This chapter also dicusses the concepts of implied volatility, flat volatilityand forward volatility in the context of two specific models, the Ho-Lee model and the Black, Derman, and Toy model. Traditionally, however, the duration is not defined against the continuously compounded interest rate but rather against the semi-annually compounded yield to maturity. The accrued interest is computed using the following intuitive formula: Accrued interest = Interest due in the full period × × Number of days since last coupon date Number of days between coupon payments Market conventions also determine the methodology to count days. In particular, we assume: −T − T 1 − e λ1 1 − e λ2 − λT − λT − θ2 e 1 + θ3 −e 2 (2.56) r(0, T) = θ0 + (θ 1 + θ 2) T T λ1 λ2 where the parameters to estimate are 6: θi , i = 0, .., 3 and λ1 and λ2 . EXAMPLE 3.4 Consider a 10-year, 6% coupon bond. 10 9 8 7 6 5 4 3 2 1 To Tommaso Gabriele and Sofia CONTENTS Preface Acknowledgments xix xxxiii PART I BASICS 1 AN INTRODUCTION TO FIXED INCOME MARKETS 1.1 1.2 1.3 1.4 Introduction 1.1.1 The Complexity of Fixed Income Markets 1.1.2 No Arbitrage and the Law of One Price The Government Debt Markets 1.2.1 Zero Coupon Bonds 1.2.2 Floating Rate Coupon Bonds 1.2.3 The Municipal Debt Market The Money Market 1.3.1 Federal Funds Rate 1.3.2 Eurodollar Rate 1.3.3 LIBOR The Repo Market 1.4.1 General Collateral Rate and Special Repos 1.4.2 What if the T-bond Is Not Delivered? The only difference from before is that we now have to discount the amount 101 not back to t = 0 at the rate r 2 (0) = 2%, but back to t = 0.25 at the current 3-month rate. Commercial paper quotes; 7. Finally, a similar growth occurred in corporate debt, which next to U.S. debt was comparatively small in 1985, but grew steadily over the years, to reach $6.2 trillion by December 2008. We tackle the former problem in this section, and the latter in the next. Chapters 5 and 6 describe these contracts and delve into the differences between futures and forwards. A savings and loan earns a large part of its revenues from the difference between the long-term mortgages it provides to home owners and the short-term deposit rate it offers to depositors. What about the coupon c(1) at maturity T = 1? Library of Congress Cataloging-in-Publication Data: Fixed Income Securities / Pietro Veronesi Printed in the United States of America. Treasury bills are quoted on a discount basis. cn (Tn) ⎞ ⎟ ⎟ ⎟ ⎠ Essentially, each row i of C corresponds to the cash flows of bond i for all the maturities T1 ,...,Tn . This example shows the following: Fact 2.2 For a given interest rate figure (e.g. 5%), more frequent accrual of interest yields a higher final payoff. The price at which the security is bought back is greater than the selling price and the difference implies an interest rate called the repo rate. We say that the trade implies a positive carry if the interest on the bond is above the repo rate and negative carry if the interest on the bond is below the repo rate. You are standing on February 15, 1994 and you hold the following portfolio: (a) What is the total value of the portfolio? In conclusion, the value of the inverse floater is given by Price inverse floater = Pz (0, 3) + Pc (0, 3) − PF R (0, 3) (2.44) = $87.45 + $128.83 − $100 (2.45) = $116.28 (2.46) 2.8.4 Leveraged Inverse Floaters Within Orange County's portfolio there were many different types of inverse floaters (e.g. different maturities and maximum interest rates). Definition 3.7 The expected shortfall is the expected loss on a portfolio P over the horizon T conditional on the loss being larger than the (100 − α)%, T VaR: Expected shortfall = E [LT |LT > V aR] (3.32) For instance, a $100 million portfolio may have a 95%, 1-month expected shortfall of $4.28 million, meaning that when a bad event hits (losses higher than VaR), the portfolio stands to lose $4.28 million in average. The notion of a parallel shift in the spot curve is always well defined for any interest rate security, and whenever an analytical formula is not available, we can always rely on computers to obtain an approximate quantity, called effective duration. Which one is more adequate for the investor's objective? What risks does it take? 2 See A Preliminary Analysis of the President's Budget and an Update of CBO's Budget and Economic Outlook, Congressional Budget Office, March 2009. In addition, the chapter contains a discussion of the pros and cons of using forwards, futures, and options for hedging purposes. I hope that my decision to have two full parts of the book requiring only a minimal analytical background will push readers to try to correctly assess the riskiness of complex fixed income securities, to see better what they are buying, and whether there is any reason why a security may appear to yield a higher-than-market return. In 1997 the U.S. government started issuing TIPS – Treasury Inflation Protected Securities – that is, securities that are indexed to inflation. The notion of the first derivative of P z (t, T) with respect to r then gives the sensitivity of the zero coupon bond to the interest rate r. Recall from Chapter 2 that inverse floaters have a coupon that moves inversely to shortterm floating rates. But if she is planning to use up some of the capital for consumption, this strategy is clearly risky. 18 The coupon is given by c(t) = 25% − 2 × r1 (t − 1) (2.47) What is a portfolio of bonds that pays this cash flow? In fact, the repo dealer typically gives something less than the market price of the bond, the difference being called a haircut. Because the term structure of interest rate is flat, a 4% coupon bond would be valued at par ($100), which is the amount the arbitrageur needs to borrow. In reality, if this situation occurs, the bond does not pay any coupon [i.e., c(t) = 0 if r(t − 1) > 15%]. Denoting by T 3 = December 31, 2006, the price of this note can be written as Pnote(t, T3) = $99.1093 = $1.5 × Z(t, T1) + $1.5 × Z(t, T2) + $101.5 × Z(t, T3) (2.25) We already determined Z(t, T1) = 0.983607 and Z(t, T2) = 0.965542 in Example 2.9. In Equation 2.25 the only unknown element is Z(t, T 3). Pricing Path Dependent Options 13.4.1 Illustrative Example: Long-Term Asian Options 463 465 466 468 469 472 473 473 CONTENTS 13.5 13.6 13.7 13.8 13.4.2 Case Study: Banc One AIRS Spot Rate Duration by Monte Carlo Simulations Pricing Residential Mortgage Backed Securities 13.6.1 Simulating the Prepayment Decision 13.6.2 Additional Factors Affecting the Prepayment Decision 13.6.3 Residential Mortgage Backed Securities 13.6.4 Prepayment Models Summary Exercises xiii 473 481 482 483 484 487 490 490 492 PART III TERM STRUCTURE MODELS: CONTINUOUS TIME 14 INTEREST RATE MODELS IN CONTINUOUS TIME 499 14.1 502 504 505 506 510 515 521 525 526 529 14.2 14.3 14.4 14.5 14.6 14.7 14.8 15 Brownian Motions 14.1.1 Properties of the Brownian Motion 14.1.2 Notation Differential Equations Continuous Time Stochastic Processes Ito's Lemma Illustrative Examples Summary Exercises Appendix: Rules of Stochastic Calculus NO ARBITRAGE AND THE PRICING OF INTEREST RATE SECURITIES531 15.1 15.2 15.3 15.4 15.5 15.6 15.7 Bond Pricing with Deterministic Interest Rate Interest Rate Security Pricing in the Vasicek Model 15.2.1 The Long / Short Portfolio 15.2.2 The Fundamental Pricing Equation 15.2.3 The Vasicek Bond Pricing Formula 15.2.4 Parameter Estimation Derivative Security Pricing 15.3.1 Zero Coupon Bond Options 15.3.2 Options on Coupon Bonds 15.3.3 The Three Steps to Derivative Pricing No Arbitrage Pricing in a General Interest Rate Model 15.4.1 The Cox, Ingersoll, and Ross Model 15.4.2 Bond Prices under the Cox, Ingersoll, and Ross Model Summary Exercises Appendix: Derivations 15.7.1 Derivation of the Pricing Formula in Equation 15.4 532 535 535 537 538 541 545 545 547 548 549 550 551 552 554 559 559 xiv CONTENTS 15.7.2 15.7.3 16 The Derivation of the Vasicek Pricing Formula The CIR Model 560 561 DYNAMIC HEDGING AND RELATIVE VALUE TRADES 563 16.1 16.2 16.3 The Replicating Portfolio Rebalancing Application 1: Relative Value Trades on the Yield Curve 16.3.1 Relative Pricing Errors Discovery 16.3.2 Setting Up the Arbitrage Trade Application 2: Hedging Derivative Exposure 16.4.1 Hedging and Dynamic Replication 16.4.2 Trading on MisPricing and Relative Value Trades The Theta - Gamma Relation Summary Exercises Case Study: Relative Value Trades on the Yield Curve 16.8.1 Finding the Relative Value Trade 16.8.2 Setting Up the Trade 16.8.3 Does It Work? 2.4.1.1 A No Arbitrage Argument We can establish Equation 2.13 also by appealing to a no arbitrage argument. In well-functioning markets we should not expect arbitrage strategies to persist for a long time. Column (5) indicates the duration of the annuity. For instance, if the Federal Reserve drops the Fed funds rate, then we may expect all of the short-term interest rates to fall. Traders refer to it as the "price value of a basis point", or PVBP, or PV01: Definition 3.5 The price value of a basis point PV01 of a security with price P is defined as (3.26) Price value of a basis point = P V 01(orP V BP) = −D P$ × d r 3.2.8 Duration and Value-at-Risk Value-at-Risk (VaR) is a risk measure that attempts to quantify the amount of risk in a portfolio. The discount factor of longer maturities is lower than the one of shorter maturities, as Fact 2.1 would imply. Similarly, in Chapter 2 I describe floating rate notes, not only because I can then use this concept to illustrate the pricing of inverse floaters (a case study at the end of the same chapter), but also because in Chapter 5 I use the same concept to describe the pricing of swaps, which is the largest fixed income market by notional amount (about $350 trillion at the end of 2008). In this sense, the notion of "duration" as a temporal average of cash flows plays no role here, as we are interpreting the duration as the sensitivity of the security's price to changes in interest rates. International Swap and Derivative Association (ISDA): This web site contains very useful information about OTC derivative markets, and the OTC market practice. 3.7.5 What Can We Infer about the Orange County Portfolio? Similarly, the academic literature has uncovered numerous stylized facts about the time variation of yields, which I also briefly summarize in a chapter. The rate of interest corresponds instead to the (annualized) rate of return on the investment within the compounding period, but it differs from it otherwise. However, if interest accrues, say, every 6 months, then the correct amount at maturity would be 5% 5% × 1+ = $105.0625, ($100) × 1 + 2 2 which is higher than $105. Sometimes it is possible to use the bonds that expire a few days earlier or later than the ones in the six-month cycle needed for the bootstrap. A first idea might be to find zero coupon bonds for all these periods. Just as important, we will focus on the impact that no arbitrage has on the riskof fixed income instruments and therefore their risk management. This fact suggests that either there is a trading opportunity available, or that that data point is an aberration and should be corrected. Chapters 3 and 4 discuss the types of risk embedded in fixed income securities, the issue of risk measurement, as well as the practice of risk management, such as asset-liability management and immunization strategies. However, as we will discuss in more detail in Chapter 5 and elsewhere, the growth in the swap THE DERIVATIVES MARKET 25 market came about because of the extreme usefulness of swaps as a convenient means for cash management and risk management. It does not cost anything at initiation, and it generates a sure positive profit by a certain date in the future; 2. That is, market participants were willing to exchange 0.97477 dollars on the first date for 1 dollar six months later. Definition 3.3 Let A and a be two constants and x be a variable. We then have E [LT |LT > V aR] = = −E [dP |dP < −V aR] = −E [dP |dP < (μP − σ P × z)] dP − μP dP − μP − μP + σ P × E | < −z)(3.55) σP σP 112 BASICS OF INTEREST RATE RISK MANAGEMENT The quantity X = dP −μ P σP is a standardized normal distribution. Translating the discount factors into annualized interest rates provides a better sense of the relative value of money across maturities. 3.9 APPENDIX: EXPECTED SHORTFALL UNDER THE NORMAL DISTRIBUTION In this appendix we derive the formula in Equation 3.34 for the expected shortfall under the normal distribution case. 2 Since the payoff at T is known at t, the relation between investment today at t and the payoff at T defines the time value of money, and Z(t, T) given in Equation 2.2 defines the rate of exchange between money at T and money at t. The tails of the probability distribution matter for risk. This chapter shows that the notion of duration is an incomplete xxiv PREFACE measure of risk, and relatively simple modifications to the model allow for a much better hedging performances, especially through the notion of factor neutrality. The stripped coupon are the zero coupon bonds that are created only from the coupon interest payments of Treasury notes and bonds. The best way to understand a repo transaction is to consider it as collateralized borrowing. As with Treasury notes, bonds also carry a semi-annual coupon. c × 100 2×(T −T2) × (1 + y/2) 2 c × 100 2×(T −Tn−1) × (1 + y/2) 2 c × 100 + + 100 2 n c × 100 2×(T −Ti) (1 + y/2) + 100 × 2 + = i=1 We can now compute the present value of the total payoff at T , using y as the constant semi-annual yield and thus Z y (t, T) = (1 + y/2)−2(T −t) as the discount factor. These rates correspond to the average interest rate that banks charge to each other for short-term uncollateralized borrowing in the London market. 18 We maintain the assumption that the coupon is always positive, that is that rates are always below 25%/2 66 BASICS OF FIXED INCOME SECURITIES 2.9.1 Bootstrap Again The iterative procedure described in the text is simple, but cumbersome. For the 1-year Treasury bill also compute the semi-annually compounded yield. • Short $20 million of 4-year floating rate bonds with a 45 basis point spread paying semiannually • Short $30 million of a 5-year zero coupon bond 7. In this case, however, the discount function lacks the necessary stiffness to avoid being contaminated by small errors in data. The no arbitrage condition requires that no arbitrage opportunities exist. For instance, a 95%, one month VaR provides the maximum loss a portfolio sustains with 95% probability. Another far more common possibility is to use derivative securities, such as swaps, to alter the duration of assets. Money markets and money markets rates: This market is the source of short-term borrowing by financial and non financial institutions. + 1 + 2 2 2 COUPON BONDS 45 then the arbitrageur can buy the coupon bond for P c(t, Tn) and sell immediately c/2 units of zero coupon bonds with maturities T 1 , T2 ,..,Tn−1 and (1 + c/2) of the zero coupon bond with maturity T n . The cost for failing to deliver for the trader was simply to forgo the repo interest. To perform the bootstrap analysis, we need many fewer bonds. The notion was that if a spread exists because of the risk of default, it should be the same across floating and fixed coupon bonds. Such an occurrence is called a fail, and up to May, 2009 such a failure to deliver would have simply implied that the repo dealer in this example would have kept the cash received, Pt , plus the repo interest. Such situations may involve the pricing of some structured derivatives, or their risk assessment using some measures of risk, or describe an arbitrage trading situation and the risk involved in carrying it out. The discount factor curve Z(0, T) is in the fourth column. Clearly, the safer strategy is in the middle. In fact, for the same reason that investors value $1 today more than $1 in six months, they also value $1 in three months more than $1 in six months. In Part II, the exercises require spreadsheet programs to build binomial trees that fit real-world fixed income securities, such as bonds, swaps and options. Lower Average Time of Cash Flow Payments: The higher the coupon, the larger are the intermediate coupons relative to the last one (in 10 years). Arbitrage strategies: These are strategies that cost nothing to enter into, and provide sure money within a certain time. Source: Federal Reserve Bank of New York obtained from SIFMA web site: Total 112.1 170.3 184.1 245 320.8 452.2 567 623.3 781.4 790.5 913.6 1,139.3 1,359.7 1,477.1 1,440.3 1,691.8 2,042.0 2,525.5 2,431.1 2,532.9 3,097.7 3,788.1 4,041.1 4,946.7 5,643.6 5,613.5 19 20 AN INTRODUCTION TO FIXED INCOME MARKETS Figure 1.6 Short-Term Rates: 1991 - 2008 Panel A. No warranty may be created ore extended by sales representatives or written sales materials. Although such an investment per se appears very safe, the presence of the large leveraged position generates a much higher duration of the leveraged portfolio itself, which could lead to the $1.6 billion losses when the interest rates moved by 2.7%, as they did in 1994. For now, we turn to describing individual markets in more detail. Is an investment in long-term U.S. Treasury bonds really safe? For example, consider a 3-year leveraged inverse floater that pays a coupon of 25% minus two times the short-term interest rate. If you do not find everything on Wiley web site and/or you are interested in receiving some material in Latex / PDF, please let me know by email at pietro.veronesi@ChicagoBooth.edu. That is, the expected monthly loss of the Orange County portfolio in case of an extreme event is $990 million. Repo and reverse repo rates; 5. The panels in Figure 94 BASICS OF INTEREST RATE RISK MANAGEMENT Table 3.3 Example of Immunization Strategy (1) (2) (3) t r2 Wt 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0 8.5 9.0 9.5 10.0 10.5 11.0 11.5 12.0 12.5 13.0 13.5 14.0 14.5 15.0 15.5 16.0 16.5 17.0 17.5 18.0 18.5 19.0 19.5 20.0 20.5 21.0 21.5 22.0 22.5 23.0 23.5 24.0 24.5 25.0 25.5 26.0 26.5 27.0 27.5 28.0 28.5 29.0 29.5 30.0 4.00% 4.53% 5.46% 5.80% 5.07% 5.70% 5.97% 5.51% 5.75% 5.62% 5.41% 4.44% 3.84% 4.37% 4.85% 5.22% 5.80% 6.21% 7.10% 7.90% 8.77% 8.00% 8.34% 7.91% 7.96% 8.59% 9.55% 9.27% 10.09% 10.49% 10.19% 10.10% 10.84% 11.34% 12.08% 11.95% 11.58% 11.45% 11.63% 11.94% 11.82% 11.69% 11.35% 11.48% 11.35% 10.98% 10.15% 10.30% 10.72% 10.32% 9.51% 9.97% 10.15% 9.54% 10.38% 10.79% 10.12% 9.64% 8.99% 8.30% 1,000,000.00 931,694.92 834,369.76 798,630.50 858,995.30 798,169.63 770,555.63 802,061.06 775,789.15 780,072.25 789,534.15 863,655.83 909,537.36 851,902.96 803,888.62 767,558.29 720,606.80 688,142.41 633,131.27 589,160.97 547,447.66 574,852.24 555,509.40 567,397.43 559,108.10 529,197.45 491,704.85 495,447.31 465,514.05 449,693.51 452,268.71 448,755.10 425,079.56 408,561.98 388,211.13 385,320.32 386,506.22 382,431.96 372,563.61 360,381.56 354,983.36 349,068.99 345,393.43 334,579.86 326,472.31 320,443.04 318,131.85 303,990.10 287,314.17 276,993.67 268,191.16 249,375.96 232,116.30 217,795.73 196,476.08 176,812.76 158,948.97 138,908.31 117,422.88 94,234.40 69,375.15 (4) PV Annuity (5) D Annuity (6) PV T-bond (7) D T-bond (8) 1,000,000.00 931,527.35 833,021.75 797,418.94 855,333.53 794,486.32 767,029.58 797,212.37 771,099.47 774,970.46 783,823.66 853,748.21 896,984.46 839,884.97 792,339.74 756,355.88 709,848.70 677,682.96 623,058.85 579,458.13 538,080.12 563,207.45 543,902.64 554,342.75 545,629.81 516,057.51 479,048.80 481,641.25 452,081.69 436,142.65 437,477.20 433,060.02 409,594.85 392,937.71 372,656.40 368,625.65 368,186.27 362,838.04 352,126.89 339,248.28 332,387.26 324,921.21 319,247.45 307,161.34 297,276.29 288,950.85 283,432.61 267,790.08 249,904.43 236,955.12 224,811.75 204,603.79 185,357.01 167,873.19 145,132.50 123,208.88 101,873.37 78,608.20 53,876.71 27,622.23 12.35 11.84 11.12 10.79 11.08 10.59 10.32 10.43 10.17 10.10 10.05 10.34 10.45 10.04 9.68 9.37 9.00 8.71 8.27 7.90 7.52 7.61 7.39 7.36 7.21 6.93 6.59 6.52 6.23 6.04 5.95 5.83 5.59 5.39 5.16 5.04 4.94 4.81 4.64 4.46 4.32 4.16 4.02 3.84 3.67 3.51 3.35 3.15 2.94 2.75 2.56 2.34 2.12 1.91 1.68 1.45 1.22 0.98 0.74 0.50 1.00 0.91 0.79 0.75 0.84 0.77 0.74 0.79 0.76 0.78 0.81 0.93 1.03 0.95 0.88 0.84 0.78 0.74 0.66 0.61 0.55 0.61 0.59 0.62 0.62 0.59 0.54 0.56 0.52 0.51 0.53 0.54 0.51 0.50 0.48 0.49 0.52 0.53 0.53 0.53 0.55 0.57 0.59 0.60 0.62 0.65 0.70 0.71 0.71 0.74 0.78 0.79 0.80 0.84 0.84 0.85 0.89 0.92 0.95 0.98 1.00 17.73 17.00 15.88 15.42 16.03 15.31 14.94 15.24 14.90 14.89 14.92 15.51 15.76 15.19 14.69 14.28 13.75 13.35 12.70 12.14 11.57 11.84 11.56 11.62 11.45 11.05 10.55 10.52 10.11 9.86 9.80 9.67 9.33 9.06 8.74 8.60 8.49 8.32 8.08 7.82 7.60 7.37 7.13 6.84 6.56 6.26 5.97 5.62 5.25 4.89 4.51 4.11 3.70 3.28 2.84 2.39 1.94 1.47 0.99 0.50 0.70 0.70 0.70 0.70 0.69 0.69 0.69 0.68 0.68 0.68 0.67 0.67 0.66 0.66 0.66 0.66 0.65 0.65 0.65 0.65 0.65 0.64 0.64 0.63 0.63 0.63 0.62 0.62 0.62 0.61 0.61 0.60 0.60 0.59 0.59 0.59 0.58 0.58 0.57 0.57 0.57 0.57 0.56 0.56 0.56 0.56 0.56 0.56 0.56 0.57 0.57 0.57 0.58 0.59 0.60 0.63 0.67 0.75 1.00 xt (9) Interest Payment 6,069.48 6,394.58 6,818.28 6,949.54 6,727.23 6,997.57 7,111.90 6,975.91 7,094.48 7,054.71 6,967.50 6,387.38 5,880.42 6,309.03 6,648.02 6,882.78 7,219.58 7,429.60 7,834.29 8,132.71 8,401.06 8,227.02 8,354.68 8,481.00 8,813.42 8,733.50 9,009.18 9,135.49 9,046.62 9,002.83 9,243.38 9,387.23 9,600.17 9,532.93 9,356.17 9,237.02 9,218.34 9,242.82 9,072.49 8,874.41 8,556.37 8,426.27 8,158.47 7,745.83 7,078.31 6,876.17 6,770.09 6,252.49 5,521.92 5,359.27 5,027.23 4,343.35 4,180.27 3,771.06 2,981.69 2,210.22 1,308.69 0.00 (10) Coupon T-bond 13,930.52 14,198.57 14,817.53 14,901.35 14,125.61 14,430.69 14,430.83 13,867.73 13,838.82 13,518.96 13,154.64 12,332.29 11,770.13 11,906.61 11,997.12 12,008.02 12,133.76 12,138.47 12,422.01 12,628.90 12,856.82 12,148.50 12,038.36 11,532.52 11,269.64 11,279.49 11,424.64 10,977.17 11,013.67 10,837.50 10,380.01 10,010.52 9,931.53 9,743.14 9,615.89 9,203.05 8,733.27 8,340.71 8,024.12 7,733.61 7,350.17 6,973.10 6,571.90 6,249.34 5,895.98 5,523.33 5,123.72 4,825.62 4,545.18 4,213.72 3,875.08 3,603.08 3,320.09 3,021.95 2,763.55 2,501.20 2,242.73 2,016.28 1,852.74 1,924.37 INTEREST RATE RISK MANAGEMENT 95 Figure 3.4 Performance Immunization Strategy in Simulations 900 800 Number of Events in 10000 Simulations 700 600 500 400 300 200 100 0 0 100,000 200,000 Dollars 300,000 400,000 96 BASICS OF INTEREST RATE RISK MANAGEMENT Figure 3.5 Performance Fixed Investment Strategy in Simulations A. Consider a 10-year coupon bond paying 6% coupon rate. How can derivatives help hedge risks? The first three columns of Table 3.2 show this effect for the case in Example 3.4. What is the intuition? In the VaR case, we first rank all of the portfolio P&L realizations under the various interest rate scenarios from the worst to the best, and then pick the 5% worst case. If the interest rate goes to zero, for instance, then there is not enough of capital to make up the annuity coupon. The two effects, called "cash flow effect" and "discount effect," work in opposite directions. Why? The swap market, for instance, which we still call a derivative market and whose xxii PREFACE size at the beginning of the 1990s was negligible, now has a global market value of over $8 trillion, and a notional of over $350 trillion. For this reason, the book is filled with real-world examples and case studies, as discussed below. Board of Governors of the Federal Reserve System: Research Data. This is the course I regularly teach to the second year MBA students at the University of Chicago Booth School of Business, and it is also appropriate for students enrolled in master in finance programs. The financial crisis of 2007 - 2009 generated a 'flight-to-quality,' meaning that investors dumped all risky securities and strongly demanded safe U.S. Treasuries, as the demand for safe collateral increased. 87 DURATION That is: dr ∼ N (μ, σ 2) == dP ∼ N (μP , σ2P) (3.30) The 95% VaR is then given by 95% VaR = −(μP − 1.645 × σ P) (3.31) where −1.645 corresponds to the 5-th percentile of the standard normal distribution, that is, if x ∼ N (0, 1) then P rob(x < −1.645) = 5%. Three-year zero coupon bond. 3 The expected inflation series is computed as the predicted annual inflation rate resulting from a rolling regression of inflation on its 12 lags. Francisco Javier Madrid and Nina Boyarchenko provided precious help with some exercises and case studies, and I thank them for this. The corresponding expected shortfall, obtained by averaging all of the portfolio losses that are lower than $715 million, turns out to be $990 million. Assuming the investor will hold the bond until maturity, computing the expected return on an investment in a zero coupon bond is relatively straightforward, as the final payoff is known and there are no intermediate cash flows. Where is this number coming from? Fixed Income Securities. Assuming the remaining part of the portfolio was invested in safe Treasury securities, what should have the duration of this additional investment been? The concept of "line up" will become clear in future chapters. Thus, the cum-coupon value of the floating rate bond at the next reset date is PFCR (0.5, T) = 100 + c(0.5) = 100 + 100 × r2 (0)/2 Therefore, the value at time 0 < t < 0.5 of this cash flow is PF R (t, T) = Present value of PFCR (0.5, T) = Z(t, 0.5) × 100 × [1 + r2 (0)/2] We summarize these results in the following: Fact 2.12 Let T1 , T2 , ... In particular, if today is coupon date (but the coupon has not been paid yet), the duration is zero. Figure 3.1 Zero Coupon Bond Yields and the Level of Interest Rates: 1965 - 2005 A: Term Structure of Interest Rates Interest Rate (%) 20 15 10 5 0 1965 1970 1975 1980 1985 1990 1995 2000 2005 1995 2000 2005 B: The Average Level of Zero Coupon Yields Interest Rate (%) 20 15 10 5 0 1965 1970 1975 1980 1985 1990 Data Source: CRSP. The coupon at time t = 0.5 depends on today's interest rate r 2 (0), which is known. Note that both the present value and the duration of the annuity tend to decline over time. In particular, we have 9 Price T-note c=4.750 = = Price T-bond c=8.875 = 4.750 × 2 9.5 Z(0, T) + 100 × Z(0, 9.5) T =0.5 107.8906 (2.31) 9.5 8.875 Z(0, T) + 100 × Z(0, 9.5) × 2 T =0.5 = 141.5267 (2.32) What are the yield to maturity of these two securities? Because borrowing is collateralized, it is considered a safer way to lend cash, and this contributed to its growth over the years, making the repo market one of the most important sources of financing for traders. We compute these risk measures under both the historical distributionand normal distributionapproach by making use of all the information available up to January 1994. The standard normal distribution is plotted along with the (renormalized) histogram in Panel C of Figure 3.3. The 95% VaR is then equal to 95% VaR = −(μ P − 1.645 × σ P) = $3.4194 million. Software There are numerous examples in the book which use real-world data to illustrate the concepts discussed in each chapter. The quotation marks around the word "safe" underly an important caveat, though, which is what makes the analysis of fixed income securities so interesting. A case study at the conclusion of the chapter illustrates the activities of the Federal Reserve by using the financial crisis of 2007 - 2008 as an example. Column (2) reports a possible path of interest rates, from 4% to 11% and down again to 8% in the course of 30 years. Which convention is used depends on the security considered. 4 We renormalized the histogram to make it comparable with the normal distribution case, discussed in the next point. EXAMPLE 3.8 Consider again Example 3.7. The 95%, 1-month expected shortfall is easily computed in the case of a normal distribution, as we must simply change the coefficient "1.645" that multiplies σ P with the coefficient "2.0628" (and similarly for the 99% expected shortfall). We can also use some assumption about the distribution of interest rates. We present more details in Chapter 7. In this section we make the relation explicit. For instance, if the quarterly compounded 3-months rate is also 2%, that is r 4 (0.25, 0.5) = 2%, then Value bond at 0.25 = Present value of $101 = $101 = $100.4975 (1 + 0.02/4) In this case, the value of the bond depends on the current interest rate. We use such models to link different types of instruments by no arbitrage and therefore establishing the price of one, perhaps complex, security by using the price of a more primitive security. A related risk concerns the fact that the coupon and principal payments are in U.S. dollars, which may entail a currency risk for an overseas investor. Dollar duration: Unlike duration, the dollar duration measures the (negative of the) dollar changes in prices due to a parallel shift in the term structure of interest rates. At every other maturity T i the arbitrageur has a zero net position, as he receives the coupon from the Treasury and turns it around to the investors to whom the arbitrageur sold the individual zero coupon bonds. Individual municipalities also issue debt independently to finance local projects. At time t the trader and the repo dealer agree that the trader will return back the amount borrowed, (P t − haircut), plus the repo rate. 2.2.1 Discount Factors, Interest Rates, and Compounding Frequencies The examples above illustrate that discount factors and interest rates are intimately related, once we make explicit the compounding frequency. One drawback of this strategy, though, is that the financial institution should find exactly the type of securities that are required for the cash flow matching, such as the sequence of zero coupon bonds with $28,500 face value at 6-month intervals. What if the interest rate at time t = 0.5 was r2 (0.5) = 6%? Let F (x) = A × e ax be a function of x. Thus, we have PF R (3) = $100. 2 Basics of Fixed Income Securities. A case study at the end of the chapter also demonstrates how we can measure the duration and convexity of MBSs (and other securities) by using data instead of pricing formulas. The profit to the trader is then P T − Pt −Repo interest. Consider now an interest rate increase of 3% from 4% to 7% (as happened, for instance, in 1994). These are short-term deposits, ranging from 3 months to one year. Histogram 120 Number of Events 100 80 60 40 20 0 −4 −3 −2 −1 0 1 Change in the Level of Interest Rates (%) 2 3 Data Source: CRSP. The only term left to calculate is the duration of the fixed rate bond D F ixed . Methodologies for the computation of VaR are many and range from highly complex to fairly simple. For instance, Treasury bills use actual/360 while Treasury notes and bonds use the actual/actual counting convention. We can substitute this value into the second equation, and solve for Z(t, T2) to obtain: Z(t, T2) = $99.2343 − $1.375 × Z(t, T1) $99.2343 − $1.375 × 0.983607 = = 0.965542 $101.375 $101.375 $101.375 The prices of coupon bonds, then, implicitly contain the information about the market time value of money. We consider here the simple case discussed in Chapter 2, c = 15%, T = 3 and coupon payments are annual. 3.2.1 Duration of a Zero Coupon Bond We are now in the position of computing the duration of a zero coupon bond. In this case, we obtain: (Historical distribution approach): 95% ES = $5.0709 mil; 99% ES = $9.3344 mil (3.36) We note in particular that the 99% expected shortfall is substantially larger under the historical distribution approach than under the normal distributionapproach. These securities were also relatively simple, as the U.S. government mainly issued bonds paying a fixed amount of money semi-annually. We note that this reasoning is the one that stands behind the law of one price, introduced in Fact 1.1 in Chapter 1, the fact that securities with identical cash flows should have the same price. On the horizontal axis we have time to maturity m = T − t for m that ranges from 3 months (m = 0.25) to 10 years (m = 10) (the letter "m" stands for "maturity"). This inflation risk must be taken into account in the analysis of Treasury securities. Date: September 18, 2007. The concept of an interest rate, however, is also more complicated, because it depends on the compounding frequency of the interest paid on the initial investment. United States Department of the Treasury: This web site contains information and links about US Treasury, including Treasury auctions, the US Federal Budget, and the Congressional Budget Office Projections. This is of course an abstraction, but a useful one. One possibility is to issue long-term debt to increase the duration of liabilities. The unexpected hike in interest rates together with an aggressive leveraged investment portfolio were the main causes of the debacle. In formulas, let L T = −(PT − P0) denote the loss of a portfolio over the horizon T (a profit if negative). Indeed, independently of the level of the interest rate r2 (T1), we find that the value of bond at t = 0.5 is always equal to 100: Value bond at 0.5 = = Present value of (100 + c(1)) = 100 + 100 × r2 (0.5)/2 1 + r2 (0.5)/2 100 × (1 + r2 (0.5)/2) 1 + r2 (0.5)/2 100. At reset dates, Z(Ti , Ti+1) = 1/(1 + r2 (Ti)/2), which implies PF R (Ti , T) = 100 (2.41) It may be useful to note that although between coupon dates the value of a floating rate bond depends on the interest rates, its sensitivity to variation in interest rate is very small, as we shall see more fully in later chapters. In particular, we talk about monetary policy, economic growth, and inflation. In six months, the two securities generate exactly the same cash flow and therefore the trader is hedged: Whatever he or she receives from Security 1 is then given to the holder of Security 2. Thus, the present value as of t = 1 of the total cash flow at time t = 1.5 (i.e., coupon plus value of bond at t = 1.5) is equal to 100, again. These cash payments are affected by numerous events, discussed further in Chapter 8, which make them risky for the bank. THE GOVERNMENT DEBT MARKETS 11 Table 1.3 U.S. Treasury Debt Securities Name Maturity Coupon Rate Principal Treasury Bills Treasury Notes Treasury Bonds TIPS 4, 13, 26, and 52 weeks 2, 5, and 10 years 30 years 5, 10, and 20 years None Fixed, semi-annual Fixed, semi-annual Fixed, semi-annual fixed Fixed Fixed Adjusted for inflation terms (i.e., simply U.S. dollars), if inflation increases substantially during the life of the debt instruments, these sums of money will be able to buy less of consumer goods. Finally, my aim in writing this book was also to endow anybody who is interested in fixed income markets, even readers without a strong analytical background, to understand the complexities, the risks, and the risk management methodologies of real-world fixed income markets. FLOATING RATE BONDS 53 Consider a one year, semi-annual floating rate bond. The logic is the same as in Example 2.13. What sample period is more

relevant? For instance, if dr is normally distributed, so is dP . One Month Rates Interest Rate (%) 8 6 4 2 0 Treasury Repo LIBOR 1992 1994 1996 1998 2000 2002 2004 2006 2008 2004 2006 2008 Panel B. Free access to current market data. If the medium- and long-term loans have fixed coupons, as in fixed rate mortgages for instance, then the
duration of the assets is relatively long, for instance 5 years or more. A portfolio that is long a 25% fixed coupon bond and short two floating rate bonds achieves the coupon described in Equation 2.47. She now has to decide where to invest this amount of money for the next, say, 30 years. Indeed, while Ginnie Mae has always been a government
entity, Fannie and Freddie entered conservatorship in September 2008, which implies that their own debt securities but especially their mortgage-backed securities are default free. If interest accrues every month, then the correct amount at maturity would be 5% 5% ($100) × 1 + × . Let z be the “quantile” corresponding to the VaR. Can the
investor at time t = 0.5 guess what the cash flow will be at time T = 1? In fact, through duration, we can estimate the sensitive of a portfolio to fluctuations in the interest rate. REVERSE O/N 5.15 1W 4.95 2W 4.85 1M 4.70 REPO 5.05 4.85 4.75 4.60 LIBOR FIX 1W 5.25875 1M 5.49625 2M 5.55375 3M 5.58750 4M 5.53625 5M 5.48313 6M 5.42000 1Y
5.11250 Key Rates Prime 8.25 BLR 7.00 FDTR 5.25 Discount 5.75 THE GOVERNMENT DEBT MARKETS 9 across assets prices, which in turn determine the relative prices of fixed income instruments. Given the sheer size of the MBS market, it is as important as ever to understand the pricing and hedging of such complex fixed income securities. The
figure also plots the combinations of over-the-counter (OTC) and exchange-traded interest rate options, which also grew considerably during this time frame, to reach about $100 trillion notional by December 2007, although it declined to $86 trillion by December 2008, in the midst of the 2007 - 2009 financial crisis. In particular, we can postulate a
parametric functional form for the discount factor Z(0, T) as a function of maturity T and use the current bond prices to estimate the parameters of this functional form. Consider a uniform shift of size dr across rates that brings rates to r(t, T), given by r(t, T) −→ r(t, T) = r(t, T) + dr Let the price of the security move by dP as a consequence of the
shift: P −→ P = P + dP 2 See the case study ERISK: Orange County, downloaded from the web site . In flow terms, the bank now has to pay a high rate on the deposits, but still receives a low coupon from its assets. I then teach both binomial trees (Part II) and continuous time models (Part III), more or less in the progression described in the book.
3.2.7 The Duration of Zero Investment Portfolios: Dollar Duration The definition of duration in Equation 3.1 implicitly implies that the security, or the portfolio, has nonzero value. We cover additional details regarding the repo market and its uses by market participants in a number of case studies. A substantial increase in interest rates, for instance,
will tend to lower prices of long-term bonds, and thus the investor can end up with a capital loss. For instance, an interest rate change from 4% to 1% decreases Ms. Caselli’s annual interest income from $40,000 to $10,000, a rather dramatic change. LIBOR fixes; 10. This can be used for securities or strategies that require a zero investment. A useful
starting point is to suppose that Orange County portfolio was invested only in zero coupon bonds and then find the maturity of these bonds necessary to bring a loss of $1.6 billion. (2.12) 43 COUPON BONDS The Treasury issues zero coupon bonds with maturities up to only 52 weeks. DURATION 83 Table 3.2 Duration versus Coupon Rate and Interest
Rate Coupon c Price Pc Duration D Interest Rate r2 Price Pc Duration D 0 2% 4% 6% 8% 10% 12% 61.03 76.62 92.21 107.79 123.38 138.97 154.56 10.00 8.95 8.26 7.76 7.39 7.11 6.88 1% 3% 5% 7% 9% 11% 13% 147.47 125.75 107.79 92.89 80.49 70.12 61.44 8.13 7.95 7.76 7.56 7.35 7.12 6.90 For the same reason, the duration of a coupon bond
decreases with the general level of interest rates, as we see for the last three columns of Table 3.2. In this case, the coupon rate is kept at 6%, but the semi-annual interest rate r 2 – constant across maturities – increases from 1% to 13%. I hope this book will provide the tools, even to the less mathematically oriented reader, to understand the
complexities of fixed income modern markets. 9 One Step Binomial Trees. EXAMPLE 1.3 Consider the following situation. As it can be seen, for both maturities, the safe T-bill rate is the smallest and the LIBOR is the highest of the three rates, as borrowing and lending at the LIBOR rate is riskier as the loans are uncollateralized. The data are those
contained in Table 2.6. The results of the bootstrap methodology are already contained in Table 2.7 in the form of discount function Z(0, T). As an example, on-the-run Treasury securities typically are “on special,” in the sense that the repo rate charged for collateralized borrowing is smaller than the GCR. U.S. Treasury bill prices and yields at various
maturities; 3. Second, the two firms swap coupon payments. This chapter also connects the movement of interest rates over time to real economic growth and inflation rate, as the Federal Reserve acts to keep the economy growing and the inflation rate low. In particular, the mortgage backed securities market stands as a $8.9 trillion market, a good
$3 trillion larger than U.S. debt. Indeed, to some extent, the yield to maturity y can be considered an average of the semi-annually compounded spot rates r2 (0, T), which define the discount Z(0, T). The sensitivity of inverse floaters to interest rates can be calculated through the concept of duration. ⎞ ⎟ ⎟ ⎟ ⎠ Pc (0, Tn) We then obtain a system of n
equations with n unknowns [the unknowns are the values of Z (0, T1) , ..., Z (0, Tn)] P (0) = C × Z (0) where C is the cash flow matrix: ⎛ ⎜ ⎜ C= ⎜ ⎝ c1 (T1) c2 (T1) .. P. This is both the average time of payments (there is only one), and also the percentage loss in value from an increase in interest rates. In the example, 5% = ($105 − $100)/$100 is the
rate of return on the investment. U.S. government debt has always occupied a special place in fixed income markets, mainly because it is perceived to have an extremely low probability of default. In addition, besides the change in shape, the term structure also moves up and down as a whole. In these three years of work the world of finance changed,
as many key players in fixed income markets either collapsed (e.g., the investment banks Bears Stearns and Lehman Brothers), have been acquired by the U.S. government (e.g., the two mortgage giant agencies Freddie Mac and Fannie Mae), or have been acquired by other banks (e.g., the investment bank Merrill Lynch). We can now multiply each of
these changes dr observed in the plot by −D P ×P = −5 ×100 million to obtain the variation in dP . We can denote the total cash flow paid at time Tj as ci (Tj). Indeed, Chapter 18 covers the topics of risk measurement and risk management within continuous time models: In particular, I illustrate the notion of market price of risk, the fair
compensation that a fixed income investor should expect to realize when he or she purchases a fixed income security, as well as Monte Carlo simulations for risk assessment. The duration enters the picture here, as it measures the sensitivity of bond prices to interest rate changes. A March, 2009 analysis of the CBO about the President’s budget
proposal even predicts an increase in U.S. debt held by the public to 56.8% of GDP by 2009 and up to 80% of GDP by 2019. The Nelson Siegel curve cuts through the bootstrapped curve smoothly. But then, he can compute the value of the bond at time t = 0, because the coupon at time T1 = 0.5 is known at time t = 0 as it is given by c(0.5) = 100 × r2
(0)/2 = 101. However, it is important to note that this “average” depends on the coupon level c. However, quite independent of the types of assets, it is possible to compute the duration of the overall portfolio of assets. An easier way to obtain the same result is to use the matrix notation. In addition, this formula also justifies the use of some type of
Monte Carlo simulations to price fixed income securities. Additional examples discussing the fair valuation and the risk of derivative securities are offered throughout the book, as we explore the modeling devices applied by financial institutions to price and hedge these securities. Individuals who have borrowed through adjustable-rate retail products,
such as student... View 1 excerpt, cites background Sorry, but the page you were trying to view does not exist. Thus, the use of data and computers to determine models’ parameters, and therefore to value fixed income securities, is just part of the fixed income game. If executed properly, this method generates the desired stream of cash flows.
Building on these multi-step binomial tree models, Chapter 12 investigates the pricing of American options, that is, options that can be exercised any time before maturity. (b) What is the value of the hedged portfolio? The discount factor is posited to be given by Z(0, T) = e−r(0,T)T (2.52) 68 BASICS OF FIXED INCOME SECURITIES where the
continously compounded yield with maturity T is given by 1 − e− λ T r(0, T) = θ0 + (θ 1 + θ 2) T λ − θ2 e− λ T (2.53) where θ0 , θ1 , θ2 and λ are parameters to be estimated from the current bond data. 4 These dates have been chosen also because the spot curves had different “shapes.” Traders refer to these different shapes with particular names,
which we now describe.5 Panel A of Figure 2.3 plots the term structure of interest rates on October 30, 1992. Such an endeavor may be problematic, and costly, as many securities are nonliquid. Column (9) shows the total cash obtained at the end of each six-month period t, from the investment in overnight deposits at the beginning of the period.
100% in T−bond B. Typically, the answer is risk, that is, it may be risky to set up an arbitrage strategy and carry it out. (3.38) 93 INTEREST RATE RISK MANAGEMENT Table 3.3 illustrates the strategy. Panel B makes a histogram of these changes, that is, describes the frequency of each possible change. In addition, in modern financial markets,
countries’ central banks, such the Federal Reserve in the United States, actively intervene in fixed income markets to affect interest rates in the attempt to spur real growth and keep inflation low. (c) Once we have adjusted for paid coupons, what is the change in value of the portfolio due to interest rate movements? 1 An Introduction to Fixed Income
Markets. Panel A of Figure 3.7 shows the historical changes in the average level of interest rates at the monthly frequency. c Copyright 2011 by John Wiley & Sons, Inc. The assumption of the annual payment frequency for the inverse floater is made for simplicity, so that the calculations are easier to follow. Since equity E is given by total assets (A)
minus total liability (L), E = A−L we have that duration mismatch occurs whenever DE = 0. Balances at the Federal Reserve yield a small rate of return, which was in fact zero until September 2008. The chapter ends with a case study discussing the risks embedded in a popular trade, a swap spread trade, a case that also provides further
understanding of the swap market itself. This is an especially important problem when the underlying risk factor has a fat-tailed distribution, as shown in Figure 3.3, or when the portfolio contains highly nonlinear derivative securities, as we will see in later chapters. This market is very large in size, and although considered a derivative market, its
sheer size makes it equivalent to a primary market, in the sense that the prices of swaps are really not derived from those of other securities, but rather they depend on the relative size of demand and supply of these contracts by market participants. It has been very exciting to write a text on the risk and return of fixed income securities, and their
derivatives, in the middle of what many consider the biggest financial crisis since the Great Depression. I introduce the concept of a Brownian motion by relying on the intuition developed on binomial trees, namely, as a limiting behavior of rates as the time-step in the binomial tree converges to zero. Given that in modern times the notion of duration
is mainly used for risk management purposes, and in particular to compute the sensitivity of a security to parallel shifts in the term structure, we must be careful in interpreting duration as an average time of future payments, as this interpretation only holds for securities with fixed cash flows. Panels B - D of Figure 2.3 plot the shape of the term
structure on other occasions. Federal funds rate quotes; 2. The repo market plays an important role in the fixed income industry, as it is used by traders to borrow and lend cash on a collateralized basis. At times, it will be convenient to focus on discount factors and at other times on interest rates, depending on the exercise. That is, their coupon rate
is zero. This makes it very simple to compute, indeed. This shape is called hump and, in the example, took place on March 31, 2000. Both firms then gain from entering this deal. The early swaps managed to arbitrage some relative price discrepancy that existed between floating rates and fixed rates. 2.4.3 Expected Return and the Yield to Maturity
How can we measure the expected return on an investment in Treasury securities? But interest rates did increase, and the fund went down. 1.6.2 Futures and Forwards Table 1.2 also reports the quotes of futures contracts, for instance, the “90 Day Eurodollar Futures,”Eurodollar,futures the “Fed Funds Futures,” and the “10-year Treasury Note
Futures.” Futures contracts, discussed in detail in Chapter 6, are contracts according to which two counterparties decide to exchange a security, or cash, or a commodity, at a prespecified time in the future for a price agreed upon today. The discount factors can then be obtained by inverting the cash flow matrix: Z (0) = C−1 × P (0) APPENDIX:
EXTRACTING THE DISCOUNT FACTORS Z(0, T) FROM COUPON BONDS 67 2.9.2 Regressions As mentioned, we rarely have such nicely spaced data. This chapter also shows the negative convexity that is generated by the American option feature embedded in such securities. There are numerous reasons why people would like to have money today
rather than in the future. Parts I and II of the book are accessible to students familiar with basic calculus, while Part III requires a more analytical background. 42 BASICS OF FIXED INCOME SECURITIES at roughly the same time. In other words, for portfolio A the investor would spend the same on all securities except for the 7-year coupon bonds
(paying 2.5% semiannually) from which the investor will short enough to get to $50 million. Mathematically, we can express the limit of rn (t, T) in Equation 2.6 as n increases to infinity in terms of the exponential function: Fact 2.7 The continuously compounded interest rate r(t, T), obtained from r n (t, T) for n that increases to infinity, is given by
the formula Z(t, T) = e−r(t,T)(T −t) (2.7) ln (Z(t, T)) T −t (2.8) Solving for r(t, T), we obtain r (t, T) = − where “ln(.)” denotes the natural logarithm. The only thing we have to remember is Definition 3.2, and the rule of the first derivative in Definition 3.3 when applied to a zero coupon bond (Equation 3.4). The inputs of these models are the bond
prices, and the outputs are the prices of derivative securities. I carry on this discussion in Chapter 20, which illustrates the Black’s formula to price standard derivatives, such as caps, floors, and swaptions. This contract would be a good hedge for the firm against interest rate hikes, because, if the interest rate does increase above the strike rate, then
the option’s seller must pay the corporation a contractually agreed-upon cash flow, which the corporation can use to pay its own liability to its bond holders. In the old days, until the end of the 1980s, fixed income markets were dominated by government debt securities, such as United States government Treasury bills, notes, and bonds. We denote
the discount factor between these two dates by Z(t, T). To understand the risks in interest rate fluctuations, consider the following example. These explanations are related to the behavior of the U.S. economy, its budget deficit, and the actions of the Federal Reserve, as well as investors’ appetite for risk (or lack thereof). The value of these securities
is, by construction, equal to $1,000,000, as the present value in Equation 3.37 applies to this case. A book on fixed income securities must mention the impact that monetary policy has on interest rates. 10-year Treasury note futures; 13. The calculation of expected shortfall is only slightly more involved than the one of VaR. The computation of the
price of fixed-coupon bond with coupon rate equal to 25% yields a price of Pc (0, 3) = $156.41. 3 3 6 7 9 11 11 14 14 14 14 14 15 16 18 v vi CONTENTS 1.5 1.6 1.7 1.8 2 The Mortgage Backed Securities Market and Asset-Backed Securities Market The Derivatives Market 1.6.1 Swaps 1.6.2 Futures and Forwards 1.6.3 Options Roadmap of Future
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Duration 83 3.2.7 The Duration of Zero Investment Portfolios: Dollar Duration 84 3.2.8 Duration and Value-at-Risk 86 3.2.9 Duration and Expected Shortfall 89 Interest Rate Risk Management 90 3.3.1 Cash Flow Matching and Immunization 91 3.3.2 Immunization versus Simpler Investment Strategies 93 3.3.3 Why Does the Immunization Strategy
Work? In particular, if a local bank provides mortgages to a local community only, it is subject to the risk that these homeowners may all default at the same time because of local geographical factors. Chapter 19 discusses no arbitrage models, which are models similar to the ones introduced in Chapter 11 on binomial trees, but in continuous time. In
particular, an increasing yield curve does not predict future higher rates, but future higher bond returns (i.e., if anything, lower future rates). That number is the 99% monthly VaR computed using the historical distribution approach. And it is exactly this fact, that most “fixed” income securities in modern financial markets actually do not have a “fixed
income”, that makes the analysis of these debt instruments difficult. The rules of no arbitrage, however, are still key to defining some relationships that must exist AN INTRODUCTION TO FIXED INCOME MARKETS 8 BID/ASK LST/OPEN HIGH/LOW DJIA FED FUNDS 5 1/8 5 1/8 5 1/8 13479.91 5 1/36 5 1/8 5 1/8 +76.49 GNMN 6.0 GOLD 6.0 FNMA
6.0 5.6000 5.4300 5.1300 +10.36 EURO $ DEP 5.5000 5.3300 5.0300 2592.02 3M 6M 1Y CCMP -04+ +.23 SWAP RATES 3Y 4.809 5Y 4.918 10Y 5.174 FUNDS FUT SEP 95.01 OCT 95.11 NOV 95.30 DEC 95.39 JAN 95.44 FEB 95.56 10yr Note Fut CBT 109 - 20 CRUDE OIL NYM WTI 80.80 90D EUR $ FUT DEC 95.07 MAR 95.38 JUN 95.51 SEP 95.56
DEC 95.56 MAR 95.51 ‘ 3.81 4.17 4.15 Table 1.2 A Snapshot of U.S. Treasury and Money Market Rates -02 00 -01 DEALER CP 15D 5.060 30D 5.350 60D 5.400 90D 5.410 120D 5.380 180D 5.240 US T-BILL YIELD/PRICE 4W 3.88 0.12 3.82 3M 4.16 0.02 4.07 6M 4.31 0.02 4.17 S&P 500 FUT 1947 +7.20 -03 -04 -05+ -07 -19+ -.56 30Y MBS 100-24 100-
25 100-09 100-10 100-07 100-08 US BONDS YLD/BID/ASK/CHG 4 08/31/09 4.117 99-24+ 99-25 4 1/2 05/15/10 4.128 100-29 100-29+ 4 1/8 08/31/12 4.241 99-15 99-15+ 4 3/4 08/15/17 4.493 102-00 102-01 5 05/15/37 4.735 104-05 104-06 CRB 324.31 SPOT FOREX JPY 115.8000 EUR 1.3875 GBP 1.9983 CHF 1.1876 MXN 11.1030 CAD 1.0233 Source:
Bloomberg. For instance, in Example 3.7 the 99% VaR using the historical distribution approach is $5.52 million, while this figure is only $4.83 million using the normal distribution assumption. Indeed, long-term bonds, for instance, may suffer strong capital losses in response to a generalized increase in interest rates. A trader entering into a repo
transaction with a repo dealer is borrowing cash (the sale price) in exchange for the security, which is held hostage by the repo dealer. First, Firm A issues a floating rate bond at LIBOR + 3%, while Firm B issues a fixed rate bond at 12%. The Orange County case study at the end of the chapter discusses the pricing of inverse floaters, and provides a
decomposition of inverse floaters in terms of a coupon bond, a floating rate bond, and a zero coupon bond (see Equation 2.43). The following is a stylized example of common situations occurring at that time. I do not go over all of the possible fixed income securities, structured products, and derivative securities that have ever been invented. The
analysis in previous sections shows that the value of the assets drop, while the value of the liabilities does not change. End-of-chapter exercises using real-world data and real-world securities cement the important concepts. If they are different, why are they different? The duration is given by the sum of these weighted payment times, reported at the
bottom of the table: D c = 7.762. The regression methodology deals with the case in which there are too many bonds compared to the number of maturities. For instance, the financial institution may agree to pay $28,767 every six months for 30 years in exchange for the $1,000,000 deposit. Thus, reliance on the recent past would miss the probability
that in fact volatility will go up, and with it, the risk of large losses. As show in Table 1.1, the interest rate derivatives market is huge and it has been growing steadily for the past three decades, as illustrated in Figure 1.2. Chapters 5 and 6 explore these markets in more detail, and provide some early examples of uses of derivative contracts for
corporations and traders. Consider the two extremes: 1. If many traders want to undertake the same strategy of shorting that particular bond, then that bond is in high demand, and the repo rate for that bond declines below the general collateral rate. This finding is a reflection of the fat-tailed distribution that characterizes the interest rate changes,
and thus of the P&L dP , as shown in the bottom panel in Figure 3.3. In particular, extreme portfolio realizations occur more frequently than according to the normal distribution. 1.4 THE REPO MARKET The last entry in the top row of Table 1.2 reports the repo and reverse repo rates. The size of the pie that the two firms can divide through a swap is
given by the difference in comparative advantage implicit in Table 1.7. In other words, the spread on the fixed coupon is 3%(= 15% − 12%) while the spread on floating coupon is only 1%(= (LIBOR +3%) − (LIBOR +1%)). The second clear pattern is that all of the interest rates move together: They go up and down at roughly the same time. For
coupon bonds it is more complicated. These concepts are further developed in Chapter 12, which details the pricing and hedging of an important class of derivatives, called American options. Note, however, the rate of interest and the rate of return differ for a one-year horizon. More precisely, Table 2.8 carries out the calculation, and obtains the
price of the fixed coupon bond P c (0, T) = $128.83. The duration of assets, in contrast, is given by DA = − 1 −1.6 = 2.89 20.5 .027 (3.48) Of course, wA × DA = 7.90 as before. Conversely, we will see securities for which the duration is actually longer than their maturity, or securities for which the duration is negative. The chapter uses two popular
models, the Ho-Lee model and Black, Derman, and Toy model, to show the differences in pricing between different models, even when the inputs are the same. Below, we also review the more traditional notion of duration that uses semi-annually compounded yield to maturity in its definition. This is given by the following: Total payoff at T = c × 100
× (1 + y/2)2×(T −T1) 2 50 BASICS OF FIXED INCOME SECURITIES + ... 2.5.1 The Pricing of Floating Rate Bonds The pricing of floating rate bonds is simple, although the logic may appear a little complicated at first. (a) What is the value of the unhedged portfolio now? As the world of fixed income securities becomes more complex, I felt that
anybody who studies fixed income securities must be exposed more directly to this complexity. First, note that indeed on each date in the sample, the discount factor with shorter time to maturity is always higher than the discount factor with longer time to maturity. There are two ways to see this result intuitively: 1. The figure clearly shows that in
these cases, the bank stands to lose money with some probability. For instance, how safe is an investment in long-term U.S. Treasuries? And yet, with a moment’s reflection, it is actually simple. Let there be discount factors Z(t, Ti) for each date Ti . (a) Compute the discount curve Z(0, T) for T =6 month, 1 year, 1.5 years, and 2 years from the
following data: • A 6-month zero coupon bond priced at $96.80 (issued 5/15/2000) • A 1-year note with 5.75% coupon priced at $99.56 (issued 5/15/1998) • A 1.5 year note with 7.5% coupon priced at $100.86 (issued 11/15/1991) • A 2-year note with 7.5% coupon priced at $101.22 (issued 5/15/1992) (b) Once you get the discount curve Z(0, T) you
take another look at the data and you find the following 1-year bonds: i. Assume that it is now May 15, 1994 and that the yield curve has changed accordingly (see Table 3.7). Students will learn the concepts of dynamic replication and hedging. A simple example is given by the case in which dr has a normal distribution: Fact 3.6 Let dr have a normal
distribution with mean μ and standard deviation σ. As an example of the available STRIPS on a particular date, Table 1.5 reports the stripped coupons available on September 25, 2008. Value-at-Risk: VaR is a risk measure that computes the maximum losses a portfolio can sustain, within a given horizon, with a given probability. That is, the literature
has uncovered facts about the time variation of risk premia, which we should talk about in fixed income books. CASE STUDY: THE 1994 BANKRUPTCY OF ORANGE COUNTY 103 (a) How much should you go short/long on this bond in order to make it immune to interest rate changes? As the focus is on inflation, this chapter also covers the Treasury
Inflation Protected Securities (TIPS), Treasury securities that pay coupons and principal that are linked to the realized inflation rate. In this chapter we define only the terms appearing in Table 1.2. In the following chapters, we describe the relations between these markets and many others that do not appear in Table 1.2. The key concept is the
concept of no arbitrage, which is helpful to introduce right away. Neither the publisher nor author shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages. THE TERM STRUCTURE OF INTEREST RATES 41 Different dates correspond to different term
structure of interest rates. Given this distribution, we can compute the maximum loss that can occur with 99% probability. The issuance calendar of the U.S. Treasury is very dense. (b) Compute the dollar duration of the portfolio. There isn’t one right model. Definition 3.1 The level of interest rates is the average yield across maturities. EXAMPLE 2.6
On June 5, 2008, the Treasury issued 13-week, 26-week and 52-week bills at prices $99.5399, $99.0142, and $97.8716, respectively. Although other governments, such as those of the United Kingdom and Italy, also experimented with other types of debt securities whose semi-annual payments were not fixed, but rather linked to a floating index, for
instance, the inflation rate, such markets were relatively small. Computing the present value of this uncertain cash flow initially seems hard. The bank making the mortgage implicitly sold the option to the homeowner, and the option premium is embedded in the mortgage rate. 3.2.3 Duration of a Coupon Bond We can apply the result in Fact 3.3 to
compute the duration of a coupon bond. 3.2.6 Traditional Definitions of Duration We defined the duration as 1 dP (3.17) P dr where r is the continuously compounded interest rate. For instance, if a firm has n individual loans, whose values are A 1 , A2 ,...,An and their durations are D A,1 , DA,2 ,..., DA,n , then we can compute the duration of assets as
Duration of assets DA = n wA,i DA,i i=1 where Ai wA,i = n i=1 Ai Similarly, financial institutions also have very complex liabilities, as they do not finance their loans only with deposits, but also with longer-term vehicles (e.g., certificates of 98 BASICS OF INTEREST RATE RISK MANAGEMENT deposit), long-term bonds, and, of course, equity. This is
evident in Figure 2.3, as on three different dates we have three different shapes of the term structure. Each chapter’s content often contains the seeds of concepts described in future chapters. Interest rates, though, have a big advantage over discount factors when we analyze the time value of money: their units can be made uniform across maturities
by annualizing them. The two agencies need to issue these types of securities to hedge against the variation in the value of the mortgage backed securities (MBS) that they hold in their assets, a variation that is mainly due to interest rate fluctuations. As in Section 2.1, from the first equation we obtain the discount factor Z(t, T 1) = $98.3607/$100 =
0.983607. Of course, inside the family of splines there are many specifications, such as (to give some names): (a) Exponential cubic splines; and (b) B-splines. 11 For notational simplicity, in this section the six month rate is denoted by r (t) instead of r (t, t + 0.5). Starting with the next chapter, we begin to analyze each market in much more detail.
Unfortunately, such a correction is not easy if we use the bootstrap methodology. The appendix at the end of this chapter reviews some other methodologies that are widely used to estimate the discount factors Z(0, T) from coupon bonds. Invest all of $1,000,000 in 30-year T-bonds. Even so, examples are stripped down versions of much more complex
problems, and I use such examples to illustrate one issue at a time. The 95% VaR corresponds to the 5% worst case of distribution of dP . In particular, this case emphasizes that even if the average maturity of the instruments may be very low, the risk of such securities or portfolio may be very high. Then, we also cover the basic techniques to mitigate
financial risk, such as immunization and asset liability management. Simple polynomials: f (T) = T This is the simplest case, where the discount function is the Lth-order polynomial Z (t, T) = 1 + L a T =1 and the coefficients a have to be estimated to minimize the distance betweenob- n i c Tji × served prices in the data Pci (0, Ti) and the theoretical
prices P i (0, Ti) = j=1 i i Z 0, Tj , where Tj is the jth-s cash-flow date of bond i. In the plot T − t = 20, as the zero coupon bond has 20 years to maturity. 15 If we follow this strategy, however, we find that at maturity T = 3 the principal we receive from the fixed rate bond has to be used to pay for the principal of the floating rate bond. The exercises
are an integral part of the learning strategy: Most exercises are data driven and require the use of computers, either spreadsheets (for Parts I and II) or a programming software (for Part III). Compute the duration of the portfolios. For simplicity, we use the average discount Z(0, T) = 0.5 × ZA (0, T) + 0.5 × ZB (0, T) for the following calculations.
Finally, the last column reports the weighted payment times, ωi × Ti . 1 − The time value of money rises with maturity: The compensation that the Treasury has to provide investors to make them part with money today to receive money in the future, i.e., hold Treasury securities, increases the longer the investment period. When interest rates
increased at the end of the 1970s, savings and loans were receiving their fixed coupons from mortgages contracted in the past, when rates were low, but suddenly they had to pay interest on deposits at the new higher deposit rates. This web site contains large datasets related to several fixed income markets, including the daily estimation of the US
Treasury Yield Curve from 1961 to Present. This measure of risk answers the following question: How large can we expect the loss of a portfolio to be when it is higher than VaR? Then r2 (t, T) defines a discount factor as Z(t, T) = 1 1+ r2 (t,T) 2 2×(T −t) (2.2) The logic of this fact lies in the example above. For instance, if n days pass between the
two 16 AN INTRODUCTION TO FIXED INCOME MARKETS Figure 1.4 Schematic Repo Transaction time t MARKET buy bond at P t =⇒ == pay P t TRADER sell bond at P T =⇒ == get PT get the bond == TRADER REPO DEALER == n pay (Pt − haircut)× (1+repo rate × 360) deliver bond =⇒ == get Pt − haircut REPO DEALER time T = t + n days
MARKET dates, we have Repo interest = n × Repo rate × (Pt − haircut) 360 (1.1) where the denumerator “360” stems from the day count convention in the repo market. Did the immunization strategy work? The major risk for a portfolio manager is in fact to rely too much on these measures of risk, APPENDIX: EXPECTED SHORTFALL UNDER THE
NORMAL DISTRIBUTION 111 Figure 3.8 The Distribution of the Monthly P&L of the Orange County Portfolio 3.5 3 Normal Distribution Approach 99% VaR = $598 mil Probability Density 2.5 2 1.5 Historical Distribution Approach 99% VaR = $715 mil 1 0.5 0 −2 −1.5 −1 −0.5 0 Billions of Dollars 0.5 1 1.5 2 forgetting that they are fragile, in the sense
that they greatly depend on the way they are computed. The expected shortfall is much better able to capture the risk from tail events. In this section we pave the way for the discussion of risk in the next chapters by investigating the methodology to value inverse floaters. This implies that about four to five trillion dollars worth of mortgage backed
securities are now guaranteed by the U.S. government. Yet, notwithstanding the relative simplicity of these two parts of the book, the hands-on strategy, the real-world examples, the case studies, and the focus on real-world securities provide a strong foundation for the important concepts in fixed income asset pricing, from no arbitrage to risk
premia, from duration to positive and negative convexity, from risk measurement to risk neutral pricing. The discount curve at a certain time t defines instead the relation between the discount factors Z(t, T) and their time to maturity T − t. This chapter illustrates the concepts of American options, and the methodology to price them, by going
through several examples, such as Treasury callable securities, American swaptions, and mortgage backed securities. I also illustrate the concept of Ito’s lemma by relying on the convexity concepts discuss earlier in Chapter 4. Because the immunization strategy calls for equating the duration of the portfolio with the one of the annuity, it then
requires that at time 0: x0 % × 17.72 + (1 − x0 %) × 0 = 12.34 =⇒ x0 = 71% Assume that the financial institution rebalances every six months. Thus, the long-short portfolio can be written as W = Pc (0, T) − PF R (0, T) = 0 Let the duration of the fixed rate bond be 8.34. Actual/Actual: Simply count the number of calendar days between coupons; 2.
A 1-year note with 8.00% coupon priced at $101.13 (issued 5/15/1991) ii. Note too that the growing market of credit derivatives has reached a market value of $5.6 trillion, and a notional value of $42 trillion. We explore further this methodology in Chapter 5, after we cover the properties of swaps and other derivative securities. It requires the
availability of notes and bonds at semi-annual intervals. You should consult with a professional where appropriate. The chapter does PREFAC xxv not use any more mathematics than Part I does, but it is the first step into a bigger world, the world of no arbitrage term structure models. Practically, we simply need to set μP = 0 in Equation 3.31. It
starts out with $1 million, and then the balance declines as the financial institution makes coupon payments to Ms. Caselli. This material covers a full semester fixed income course for both MBA or undergraduate students. With this desire in mind, I wrote the book in a way to cover all of the important concepts in each part of the book, as each part
may require a different level of mathematical sophistication. From Equations 3.5 and 3.6, the duration of a zero coupon bond is equal to its time to maturity. In this case, J(θ0 , θ1 , θ2 , λ) = 0. A U.S. Treasury bond can be considered a “safe” investment in terms of its risk of default: As noted, the issuer will in all likelihood repay its debt to its creditors
(investors). In this book, we will rarely use the Macaulay duration, and therefore we reserve the term duration for modified duration. 1.2.3 The Municipal Debt Market The U.S. federal government issues debt to finance federal government expenses, such as health care and military expenses. 3.2.8.1 Warnings It is worth emphasizing immediately a
few problems with the Value-at-Risk measure of risk, as well as some potential pitfalls: 1. Cash Flow Matching. However, it is a common mistake to confuse a low volatility period as a safe period. If the interest rate was constant for the overall period and equal to 4%, then the final amount of wealth W T would be exactly zero. In the next chapter we
will follow up with this analysis and study the risk embedded in these securities. In particular, a Tnote with coupon c = 4.750% and a T-bond with coupon c = 8.875% were available. Indeed, Chapter 14 refers to Chapter 11 to introduce the notion of a Brownian motion as the limit of a binomial tree, as the step size becomes infinitesimal small.
Intuitively, if interest rates increase the financial institution gains from making coupon payments on its long term debt that are below the current rate. It is important to start from the no arbitrage principle to link all of these markets together. The U.S. Treasury is extremely unlikely to default on its obligations, and thus the relation between purchase
price and payoff of U.S. Treasury securities reveals the market time value of money, that is, the exchange rate between money today and money in the future. In Chapter 2 we cover the basics of fixed income instruments, that is, the notion of a discount, of an interest rate, and how we compute the fair valuation of Treasury bills, notes, and bonds. Let
us denote by 0 the last reset date, and by t the current trading day. Risk Neutral Pricing 9.4.1 Risk Neutral Probability 9.4.2 The Price of Interest Rate Securities 9.4.3 Risk Neutral Pricing and Dynamic Replication 9.4.4 Risk Neutral Expectation of Future Interest Rates Summary Exercises A Two-Step Binomial Tree Risk Neutral Pricing 10.2.1 Risk
Neutral Pricing by Backward Induction 10.2.2 Dynamic Replication Matching the Term Structure Multi-step Trees 10.4.1 Building a Binomial Tree from Expected Future Rates 10.4.2 Risk Neutral Pricing Pricing and Risk Assessment: The Spot Rate Duration Summary Exercises RISK NEUTRAL TREES AND DERIVATIVE PRICING 381 11.1 381 381
383 385 386 Risk Neutral Trees 11.1.1 The Ho-Lee Model 11.1.2 The Simple Black, Derman, and Toy (BDT) Model 11.1.3 Comparison of the Two Models 11.1.4 Risk Neutral Trees and Future Interest Rates xii CONTENTS 11.2 11.3 11.4 11.5 11.6 11.7 12 387 387 387 392 395 397 398 402 404 406 408 413 413 416 AMERICAN OPTIONS 423 12.1 423
427 428 431 431 435 438 440 444 447 450 451 12.2 12.3 12.4 12.5 13 Using Risk Neutral Trees 11.2.1 Intermediate Cash Flows 11.2.2 Caps and Floors 11.2.3 Swaps 11.2.4 Swaptions Implied Volatilities and the Black, Derman, and Toy Model 11.3.1 Flat and Forward Implied Volatility 11.3.2 Forward Volatility and the Black, Derman, and Toy Model
Risk Neutral Trees for Futures Prices 11.4.1 Eurodollar Futures 11.4.2 T-Note and T-Bond Futures Implied Trees: Final Remarks Summary Exercises Callable Bonds 12.1.1 An Application to U.S. Treasury Bonds 12.1.2 The Negative Convexity in Callable Bonds 12.1.3 The Option Adjusted Spread 12.1.4 Dynamic Replication of Callable Bonds American
Swaptions Mortgages and Residential Mortgage Backed Securities 12.3.1 Mortgages and the Prepayment Option 12.3.2 The Pricing of Residential Mortgage Backed Securities 12.3.3 The Spot Rate Duration of MBS Summary Exercises MONTE CARLO SIMULATIONS ON TREES 459 13.1 13.2 459 461 13.3 13.4 Monte Carlo Simulations on a One-step
Binomial Tree Monte Carlo Simulations on a Two-step Binomial Tree 13.2.1 Example: Non-Recombining Trees in Asian Interest Rate Options 13.2.2 Monte Carlo Simulations for Asian Interest Rate Options Monte Carlo Simulations on Multi-step Binomial Trees 13.3.1 Does This Procedure Work? There are 224 bonds quoted. The key concepts that are
explored in Part II are then also repeated in Part III, but by using continuous time methodologies. Immunization. 1.4.1 General Collateral Rate and Special Repos Other important definitions and characteristics of repo markets are as follows: 1. In this case, we find it equal to $3 million. For instance, in the example above in which T 1 = November 9,
2006 and T2 = February 8, 2007, if Z(t, T1) was lower than Z(t, T2) = 0.97477, it would imply that investors would be willing to give up $97.477 today in order to receive $100 in six months, but not in order to receive the same amount three months earlier. Interest rate swaps, in particular, have a market value of $16 trillion, and a notional value of
$328 trillion. 6 In fact, this futures is cash settled, so the deposit does not actually need to take place. In addition, a similar table is available for the stripped principals. When a bank issues a mortgage to a homeowner, the mortgage rests on the asset side of the bank’s balance sheet. The short position is accomplished by first borrowing the bond from
a broker, typically in the repo market, and then selling it to the market. The difference between the 10-year interest rate and the short-term interest rate is about 4%. Most likely, Ms. Caselli is interested in using up some of her capital for consumption purposes. Smith (Boston University), Michael Stutzer (University of Colorado), Manuel Tarrazo
(University of San Francisco), and Russ Wermers (University of Maryland) for their comments. However, if the investor sells the bond after one year, the return may be higher or lower than the promised yield (Equation 2.29) depending on what happens to interest rates. Pedagogically, the chapters’ order already offers guidance on how to progress
with the material. Since the cash flows are the same in the future, the arbitrageur is perfectly hedged. For instance, if we invest $100 for one year at the rate of interest of 5%, we receive in one year $105, that is, the original capital invested plus the interest on the investment. Recall that by definition of yield to maturity, the price of the coupon bond
on a coupon date can be written as D=− Pc (0, T) = n c/2 × 100 100 + 2×Tn y 2×Tj 1 + y2 j=1 1 + 2 (3.18) A little algebra shows that the modified duration (M D) of this coupon bond, when defined against the yield to maturity y, is given by 1 1 dP wj × Tj = y P dy 1 + 2 j=1 n MD = − (3.19) 84 BASICS OF INTEREST RATE RISK MANAGEMENT
where 1 wj = Pc(0, T) c/2 × 100 2×Tj 1 + y2 1 , wn = Pc(0, T) 100 × (c/2 + 1) 2×Tn 1 + y2 In other words, when we use the semi-annual compounded yield to maturity y to define the modified duration of a fixed rate bond can be computed as the weighted average of cash flow maturities times an adjustment given by
(1/(1 + y2). 6 This section and the next are based on publicly available information and they are only meant to illustrate the concepts introduced in this chapter. For every bond i = 1, ..., N let Pci 0, T i = Ci ×Z (0) + εi (2.51) where εi is a random error term that captures any factor that generates the “mispricing.” These factors include data staleness,
lack of trading or liquidity. It is important to spend some time on the pricing of floating rate bonds as a similar methodology applies to numerous other interest rate securities, such as floaters and inverse floaters (see Case Study in Section 2.8) as well as derivative instruments, such as interest rate swaps (see Chapter 5).10 EXAMPLE 2.12 The Italian
Treasury regularly issues CCT (Certificati di Credito del Tesoro), which are floating rate bonds with 7 years to maturity. Examples are Treasury notes and bonds, which pay coupon semi-annually. The continuous compounding, defined below, is also important, mainly for its analytical convenience. Then the value of the coupon bond can be computed as
Pc (t, Tn) = = c × 100 c × 100 × Z(t, T1) + × Z(t, T2) + ... 6 The first pattern that we see in Figure 2.5 is that interest rates move up and down substantially. If dr is not normal, Equation 3.31 does not hold. 7 Inflation, Monetary Policy, and the Federal Funds Rate. By definition, Di = − 1 d Pi Pi d r How can we determine the duration of the portfolio?
The VaR numbers computed above are relatively small compared to the ex-post $1.6 billion loss. Not all chapters have case studies, as it depends on the topic of each chapter. xix xx PREFACE the size of available fixed income securities. Simulations 16.8.4 Does It Work? In particular, consider a security with yield to maturity y (see Section 2.4.3 in
Chapter 2). Thus, denoting again t = August 10, 2006, now T 1 = November 9, 2006, and T2 = February 8, 2007, we find that the discount factor Z(t, T 1) = 0.98739, which is higher than Z(t, T2) = 0.97477. Moreover, a violation of Relation 2.1 also generates an arbitrage opportunity, which we would not expect to last for long in well functioning
financial markets (see Exercise 1). Discuss your findings in light of the law of one price discussed in Chapter 1 2.8 CASE STUDY: ORANGE COUNTY INVERSE FLOATERS 60 BASICS OF FIXED INCOME SECURITIES Table 2.5 Bonds and Notes on March 15, 2000 Cusip Issue Date Maturity Date Name Coupon Bid Ask 912827ZE 912827ZN 912810CT
912810CU 912810CW 912810CX 912810CZ 912827F4 912827G5 912810DA 912810DC 912810DD 912810DE 912810DG 912827N8 912810DH 912810DK 912810DM 912827S8 912810DQ 912810DR 912827V8 912810DU 912827X8 912827Y5 912827Z6 9128272J 9128272U 9128273E 9128273X 9128274F 9128274V 9128275G 8/15/1990 11/15/1990
1/12/1981 4/2/1981 7/2/1981 10/7/1981 1/6/1982 5/15/1992 8/15/1992 9/29/1982 1/4/1983 4/4/1983 7/5/1983 10/5/1983 2/15/1994 4/5/1984 7/10/1984 10/30/1984 2/15/1995 4/2/1985 7/2/1985 11/15/1995 1/15/1986 5/15/1996 7/15/1996 10/15/1996 2/15/1997 5/15/1997 8/15/1997 2/15/1998 5/15/1998 11/16/1998 5/17/1999 8/15/2000 11/15/2000
2/15/2001 5/15/2001 8/15/2001 11/15/2001 2/15/2002 5/15/2002 8/15/2002 11/15/2002 2/15/2003 5/15/2003 8/15/2003 11/15/2003 2/15/2004 5/15/2004 8/15/2004 11/15/2004 2/15/2005 5/15/2005 8/15/2005 11/15/2005 2/15/2006 5/15/2006 8/15/2006 11/15/2006 2/15/2007 5/15/2007 8/15/2007 11/15/2007 5/15/2008 11/15/2008 5/15/2009 NOTE
NOTE BOND BOND BOND BOND BOND NOTE NOTE BOND BOND BOND BOND BOND NOTE BOND BOND BOND NOTE BOND BOND NOTE BOND NOTE NOTE NOTE NOTE NOTE NOTE NOTE NOTE NOTE 8.750% 8.500% 11.750% 13.125% 13.375% 15.750% 14.250% 7.500% 6.375% 11.625% 10.750% 10.750% 11.125% 11.875% 5.875%
12.375% 13.750% 11.625% 7.500% 12.000% 10.750% 5.875% 9.375% 6.875% 7.000% 6.500% 6.250% 6.625% 6.125% 5.500% 5.625% 4.750% 5.500% 100.5742 100.8906 103.8047 105.9805 107.6406 112.3945 111.9297 101.2031 99.0469 110.6680 109.5117 110.3281 112.1523 115.3086 97.1172 118.8984 124.9375 118.2969 102.8633 121.6133
117.0664 95.9844 112.0352 100.6055 101.2031 98.7500 97.4883 99.5625 96.7578 93.1328 93.7852 87.9766 92.8242 100.6055 100.9219 103.8359 106.0117 107.6719 112.4258 111.9609 101.2344 99.0781 110.6992 109.5430 110.3594 112.1836 115.3398 97.1484 118.9297 124.9688 118.3281 102.8945 121.6445 117.0977 96.0156 112.0664
100.6367 101.2344 98.7813 97.5195 99.5938 96.7891 93.1641 93.8164 88.0078 92.8555 Data excerpted from CRSP (Daily Treasuries) ©2009 Center for Research in Security Prices (CRSP), The University of Chicago Booth School of Business. Therefore, if between the purchase of 1 Data excerpted from CRSP (Daily Treasuries) ©2009 Center for
Research in Security Prices (CRSP), The University of Chicago Booth School of Business. In the same fashion as with assets, the financial institution can consider its liabilities as a portfolio and compute the duration of liabilities. Definition 2.1 The discount factor between two dates, t and T , provides the term of exchange between a given amount of
money at t versus a (certain) amount of money at a later date T . For instance, the market for Eurodollar futures and options, financial derivatives traded at the Chicago Mercantile Exchange that allow financial institutions to bet on or hedge against the future evolution of the Eurodollar rate (see Chapter 6), is among the largest and most liquid
derivative markets in the world. Clearly, LT > V aR when dP < −V aR. We will review some recent theories in Chapter 7 2.4 COUPON BONDS U.S. government Treasury bills involve only one payment from the Treasury to the investor at maturity. Chapter 5 introduces basic interest rate derivatives, such as forwards and swaps. 2.9 APPENDIX:
EXTRACTING THE DISCOUNT FACTORS Z(0, T) FROM COUPON BONDS The Orange County case study in the previous section makes it apparent that the bootstrap methodology discussed in Section 2.4.2 has limited applicability, and this for two reasons. The parameter estimates are θ0 = 0.0754, θ1 = −0.0453, θ2 = −7.3182×10−009 and λ =
3.2286. A simple example is the Treasury bill described in Table 1.3. Other zero coupon bonds are available in the U.S. market through the STRIPS program. The increase in the term structure is rather substantial: The short-term rate passed from 2.9% to 5.5%, while the long rate passed from 5.7% to 7.5%. Given a coupon rate c, Equation 2.30 shows
that there is a one-to-one relation between the price Pc of the bond and the yield to maturity y. A similar situation exists for coupon bonds. The portfolio has value of zero at time t = 0 when the trade is set up. For instance, Italy issues the CCT bond, which is an Italian Treasury debt security whose coupon rate is indexed to the six month rate of Italian
6- 12 AN INTRODUCTION TO FIXED INCOME MARKETS Table 1.4 Security 3-YEAR 9-YEAR 10-YEAR 29-YEAR 2-YEAR 5-YEAR 7-YEAR 3-YEAR 9-YEAR 29-YEAR 2-YEAR 5-YEAR 7-YEAR 3-YEAR 10-YEAR 30-YEAR 2-YEAR 5-YEAR 5-YEAR 7-YEAR 3-YEAR 9-YEAR 9-YEAR 2-YEAR 5-YEAR 7-YEAR 3-YEAR 9-YEAR 29-YEAR 2-YEAR 5-YEAR 7-YEAR 3-
YEAR 10-YEAR 30-YEAR 2-YEAR 5-YEAR 20-YEAR 3-YEAR 9-YEAR Issuance Acitivity of Bonds, Notes, and TIPS: January 15, 2009 to July 15, 2009 Term Type NOTE 10-MONTH NOTE TIPS 10-MONTH BOND NOTE NOTE NOTE NOTE 11-MONTH NOTE 11-MONTH BOND NOTE NOTE NOTE NOTE BOND NOTE TIPS NOTE NOTE NOTE 9-
MONTH TIPS 10-MONTH NOTE NOTE NOTE NOTE 11-MONTH NOTE 11-MONTH BOND NOTE NOTE NOTE NOTE BOND NOTE NOTE TIPS NOTE 10-MONTH NOTE Issue Date Maturity Date 7/15/2009 7/15/2009 7/15/2009 7/15/2009 6/30/2009 6/30/2009 6/30/2009 6/15/2009 6/15/2009 6/15/2009 6/1/2009 6/1/2009 6/1/2009
5/15/2009 5/15/2009 5/15/2009 4/30/2009 4/30/2009 4/30/2009 4/30/2009 4/15/2009 4/15/2009 4/15/2009 3/31/2009 3/31/2009 3/31/2009 3/16/2009 3/16/2009 3/16/2009 3/2/2009 3/2/2009 3/2/2009 2/17/2009 2/17/2009 2/17/2009 2/2/2009 2/2/2009 1/30/2009 1/15/2009 1/15/2009 7/15/2012 5/15/2019 7/15/2019 5/15/2039 6/30/2011 6/30/2014
6/30/2016 6/15/2012 5/15/2019 5/15/2039 5/31/2011 5/31/2014 5/31/2016 5/15/2012 5/15/2019 5/15/2039 4/30/2011 4/15/2014 4/30/2014 4/30/2016 4/15/2012 1/15/2019 2/15/2019 3/31/2011 3/31/2014 3/31/2016 3/15/2012 2/15/2019 2/15/2039 2/28/2011 2/28/2014 2/29/2016 2/15/2012 2/15/2019 2/15/2039 1/31/2011 1/31/2014 1/15/2029 1/15/2012
11/15/2018 Interest Yield Rate % % 1.5 3.125 1.875 4.25 1.125 2.625 3.25 1.875 3.125 4.25 0.875 2.25 3.25 1.375 3.125 4.25 0.875 1.25 1.875 2.625 1.375 2.125 2.75 0.875 1.75 2.375 1.375 2.75 3.5 0.875 1.875 2.625 1.375 2.75 3.5 0.875 1.75 2.5 1.125 3.75 1.519 3.365 1.92 4.303 1.151 2.7 3.329 1.96 3.99 4.72 0.94 2.31 3.3 1.473 3.19 4.288 0.949
1.278 1.94 2.63 1.385 1.589 2.95 0.949 1.849 2.384 1.489 3.043 3.64 0.961 1.985 2.748 1.419 2.818 3.54 0.925 1.82 2.5 1.2 2.419 Price per $100 CUSIP 99.944485 97.998772 99.592335 99.104142 99.94874 99.651404 99.510316 99.753523 92.968581 92.50169 99.871675 99.718283 99.689717 99.713432 99.44721 99.36198 99.853739 100.113235
99.691687 99.968223 99.970714 103.325496 98.298568 99.853739 99.529266 99.942292 99.667005 97.504473 97.456658 99.830481 99.479306 99.22194 99.871395 99.411068 99.264139 99.901394 99.667162 99.063837 99.77965 111.579767 912828LB4 912828KQ2 912828LA6 912810QB7 912828LF5 912828KY5 912828KZ2 912828KX7
912828KQ2 912810QB7 912828KU3 912828KV1 912828KW9 912828KP4 912828KQ2 912810QB7 912828KL3 912828KM1 912828KN9 912828KR0 912828KK5 912828JX9 912828KD1 912828KH2 912828KJ8 912828KT6 912828KG4 912828KD1 912810QA9 912828KE9 912828KF6 912828KS8 912828KC3 912828KD1 912810QA9 912828JY7
912828JZ4 912810PZ5 912828KB5 912828JR2 Source: U.S.Treasury Web Site accessed on July 16, 2009. Thus, DF loating = 1. Given the sequence of cash flows, which are certain in the sense that the U.S. Government is extremely unlikely to default, we could compute the value of the bond itself if we knew the discount factors Z(t, T) to apply to
each of the four dates. Exercises Each chapter contains several exercises that cover the topics discussed, and highlights additional features of real-world fixed-income securities or trading methodologies. History shows time and again that low volatility periods are followed by high volatility periods. While the cash flow matching is relatively
straightforward, it is instructive to work through an example illustrating the immunization strategy. Even if the investor does not know the cash flow at time T = 1, because it depends on the future floating rate r 2 (0.5), the investor does know that at time t = 0.5 the ex-coupon value of the floating rate bond will be 100, independently of what the
interest rate does then. Given the discount factors Z(0, T) for T = 1, 2, 3, we can compute the price of a coupon bond by applying the bond pricing formula in Equation 2.13, with the only caveat that in this exercise coupons are annually paid, and thus we do not have to divide them by 2 (as instead we do in Equation 2.13). Figure 3.8 plots the
histogram of the changes in the portfolio (i.e. the portfolio profit and loss, or P&L). SUMMARY 57 2.6 SUMMARY In this chapter, we covered the following topics: 1. We can slightly change the bootstrap methodology to deal with this problem. Very informative the latest (2009) survey on derivative usage, showing that 94% of the largest 500 firms use
derivatives. First, for short-term maturities, there are too many bonds that mature on the same day to choose from. If the interest accrues once a year, then 5% is the correct answer. These assets, which have better characteristics in terms of diversification of risk, make up the collateral on debt securities issued to individual investors, called mortgage
backed securities. If today r 2 (0) = 2%, then c(0.5) = 100 × 2%/2 = 1. In fact, ($100) × (1 + 4.939%/2) × (1 + 4.939%/2) = $105, which is indeed the payoff from the investment of $100. A popular measure of return on investment for coupon bonds is called yield to maturity, which is defined as follows: Definition 2.6 Let Pc(t, T) be the price at time t
of a Treasury bond with coupon c and maturity T . Thus, Pz (0, 3) = 100 × 0.8745 = $87.45. 11. British Bankers' Association (BBA) LIBOR Releases: This web site contains data on LIBOR fixes from the BBA. If we agree that households and investors value $1 today more than $1 in the future the question is then how much $1 in the future is worth in
today’s money. 3.3.2 Immunization versus Simpler Investment Strategies How does the immunization strategy compare to other simpler strategies, such as investing fixed proportions in the long-term T-bond and cash? 1.6.3 Options Table 1.2 does not provide any quotes for interest rate options, as we obtained this table from the BTTM screen from a
Bloomberg terminal, which only reports U.S. Treasury and money market rates. The total portfolio value is $300 million. On March 15, 2000 you obtain the data on Treasuries in Table 2.5. Compute the semiannual yield curve, spanning over 9 years, from the data using the bootstrap procedure. We can use the past changes in the level of interest rates
dr as a basis to evaluate the potential changes in a portfolio value dP . Finally, Chapter 13 illustrates a new methodology, Monte Carlo simulations, to price very complex securities on binomial trees. More specifically, Figure 3.2 plots the price of a 20-year zero coupon bond for various values of r, ranging from 0 to 15%. The market is smaller in size,
as shown in Table 1.1. This market is considered by market participants riskier, as they do not involve THE DERIVATIVES MARKET 23 Table 1.7 Borrowing rates for firm A and B Fixed Rate Floating Rate Firm A Firm B 15% LIBOR + 3% 12% LIBOR + 2% 1.6 THE DERIVATIVES MARKET Table 1.2 also reports quotes of several derivative securities.
In this sense, the notion of a discount factor is un-ambiguous. The duration of liabilities, thus, is approximately zero. We will discuss more precisely the determinants of the term structure of interest rates in later chapters. This strategy yields an inflow of money to the arbitrageur that is equal to the difference between the right-hand side and the left-
hand side of Equation 2.22. 9 1. For instance, in Chapter 1 I describe the repurchase agreement (repo) market, because in Chapter 2, which covers the present value formulas and the use of the law of one price, I can leverage on the repo market to describe how financial institutions actually carry out long-short strategies. .. 3.1.2 The Bankruptcy of
Orange County In 1994 Orange County, California, lost $1.6 billion when the interest rate unexpectedly increased from 3% to 5.7% over the course of the year. The main goal of this case is to show that the measurement itself of risk is difficult, and so the portfolio manager should always be suspicious of any risk measure, and always ask how such a
measure was calculated. Expected Shortfall: A risk measure that computes the expected losses on a portfolio, conditional on these losses being larger than VaR, expected shortfall is a measure that is better able to deal with tail events than VaR. xlix PART I FIXED INCOME MARKETS CHAPTER 1 AN INTRODUCTION TO FIXED INCOME MARKETS
1.1 INTRODUCTION In the past two decades, fixed income markets have experienced an impressive growth, both in market value and in complexity. (c) Is the value the same? For instance, if the rate of interest is 5% and it accrues semi-annually, then within a six-month period the rate of return on the investment is 2.5%, that is, from $100 we have in
six months $102.5. If we annualize this semi-annual return we obtain 5%, which corresponds to the rate of interest. In fact, we can discount each future cash flow using its own discount factor, and sum the results. As the coupon rate increases, the duration is lower. The first derivative of P z (r, t, T) with respect to r is d e−r(T −t) d Pz = 100 × dr dr
= 100 × −(T − t) × e−r(T −t) = −(T − t) × Pz (r, t, T) (3.4) To emphasize that the zero coupon bond price depends on the current interest rate r, in this section we denote it by P z (r, t, T), that is, we add r as one of the arguments in Pz (t, T). Without comprehending why yields move, students cannot have a complete understanding of fixed income
markets. Screen BTMM. It is easier because the idea of interest is closer to our everyday notion of a return on an investment, or the cost of a loan. For instance, if there are worries of an increase in future inflation, we can expect the Federal Reserve to increase the target Fed Funds rate (see Chapter 7). For instance, if the local community is highly
specialized in a particular industrial sector, and the latter goes into an economic crisis, one could expect large layoffs in that community, which in turn would increase the probabilitythat homeowners will default on their mortgage payment obligations. Inflation will affect the rate of return on nominal long-term bonds, and therefore the prices of fixed
income securities will adjust. Indeed, under strategy 1, fluctuations in interest rates imply fluctuations in the amount of money available for consumption. These securities present numerous additional risks for investors compared to Treasury securities. The spread between the rate earned on assets and the (higher) rate paid on liabilities quickly put
many savings and loans out of business. In particular, Chapter 14 introduces continuous time methodologies, the notion of a Brownian motion, and Ito’s lemma. CHAPTER 3 BASICS OF INTEREST RATE RISK MANAGEMENT Interest rates change substantially over time, and their variation poses large risks to financial institutions, portfolio managers,
corporations, governments, and, in fact, households. 2.9.3.2 The Extended Nelson Siegel Model The Nelson Siegel model works well, but it lacks the flexibility to match term structures that are highly nonlinear. Treasury bonds (T-bonds) are longer-term debt instruments with maturity of 30 years at issuance. The following example further illustrates
the concept. The knowledge of the prices of zero coupon bonds allow us to determine the discount factor Z(t, T), as described in the previous sections. 3.3.3 Why Does the Immunization Strategy Work? First, note that a coupon bond can be represented by the sequence of its cash payments. @inproceedings{Veronesi2010FixedIS, title={Fixed Income
Securities: Valuation, Risk, and Risk Management}, author={Pietro Veronesi}, year={2010} }Preface. 70 BASICS OF FIXED INCOME SECURITIES Figure 2.7 The Fit of the Nelson Siegel Model 125 data Nelson Siegel 120 Bond Price 115 110 105 100 95 0 0.5 1 1.5 2 2.5 3 Time to Maturity 3.5 4 4.5 5 2.9.4 Curve Fitting with Splines This is an
extension of the curve fitting methodology described in Section 2.9.3, with a different specification of the discount factor Z (t, T) as a function of maturity T . Normal Distribution Approach. This implies that we do not know the value of the final cash flow at time T = 1, which is equal to 100 + c(1). However, as a brief introduction, the source of the
mortgage-backed securities market is relatively simple: homeowners across the U.S. finance their homes through mortgages, issued by local savings & loans, thrifts, and other banks. 3. 110 BASICS OF INTEREST RATE RISK MANAGEMENT Figure 3.7 The Monthly Changes in the Average Level of Interest Rates Change in Interest Rate (%) Panel A.
It is only because of this guarantee, in fact, that the three agencies have been able to issue mortgage backed securities since the last quarter of 2008, whereas the private market completely dried up. In particular, we discuss first how to measure risk for fixed income instruments, by introducing the notion of duration, value-at-risk and expected
shortfall. 10. We can now multiply each of these changes dr observed in the plot by −DP × P to obtain the variation in dP . When the interest rate move from r2 (0) = 2% to r2 (0.5) it has two effects: 1. Swap rates for various maturities; 14. Today is May 15, 2000. The discount factor Z(0, 3) = 0.8745. Similarly, at maturity, the arbitrageur receives
$102.1875 million from the Treasury, and again turns it around to the investors of the last coupon. We can solve this problem by adding to the portfolio a 3-year zero coupon bond. The first is that most of the Treasury securities are nominal securities, that is, they pay coupons and principal in dollars. Chapter 11 applies the concepts described in the
previous two chapters to illustrate the no arbitrage pricing of numerous derivative securities. We cannot propose to teach students even the basics of fixed income markets without a long and careful look at the data, and without knowing how to use data to fit models. They are issued every month, except the ten-year note, that is issued less frequently.
Part I: Basics Part I of the book, Chapters 1 to 8, covers the basics of fixed income pricing, risk, and risk management. Immunization: Immunization is a strategy to make a portfolio insensitive to changes in interest rate. In addition, most of the exercises require some data analysis. I put quotation marks around the term “fixed” because nowadays most
of the so-called fixed income securities have streams of income payments that are all but fixed. What if there are additional dates? 2.3.1 The Term Structure of Interest Rates over Time As for discount factors, the term structure of interest rates depends on the date t at which it is computed. I just introduce the Black formula, and discuss the dynamics
of implied volatilities over time, and the concept of flat and forward volatility. In PREFACE xxxi the same way nowadays it is much harder to understand how the engine of a car works, compared to the past, and mechanics need to have a better knowledge than in the past about these new engines, participants in fixed income markets, whether traders,
risk managers, regulators and so on, cannot hope to use old tools to understand modern markets, as their complexity has just being increasing through time, and new tools are necessary. Finally, Chapters 19 to 22 cover more advanced models for fixed income security pricing. Table 3.9 contains the calculations. U.S. Treasury bond yields and prices
with various maturities; 6. In this section we discuss two methodologies that are based on the concept of duration: The historical distribution approach and the normal distribution approach. PREFACE xxix For Instructors The material in this book can be taught at two different levels: An introductory level and an advanced (but not very advanced)
level. Pedagogical Strategy This book employs a hands-on strategy to highlight the valuation, the risks, and the risk management of fixed income securities. Thus, we immediately find Price leveraged inverse floater = 2 × $87.45 + $156.41 − 2 × $100 (2.49) = $131.32 (2.50) This case study illustrates that we can readily apply the tools covered in this
chapter to value more complex securities, such as inverse floaters and leveraged inverse floaters. These two interpretations are equivalent for fixed rate bonds, that is, bonds that pay fixed coupons: A zero coupon bond with maturity of 5 years has duration equal to 5. I provide instead examples and methodologies that can be applied quite universally,
once the basic concepts have been understood. Melé, Yoshiki ObayashiEconomics 2018Interest rate volatility (IRV) affects a wide base of individuals, investors, companies, and even governments. The swap contract is the largest market of all. We continue with Example 3.9. EXAMPLE 3.10 The financial institutionthat took up the commitment to pay
$28,767 every six months can ensure the ability to pay by engaging in the following dynamic strategy. By using the repo market, Orange County effectively levered up the portfolio position to $20.5 billion (see Chapter 1 to review the repurchase agreements contract.) Essentially, the Treasurer of Orange County could pledge the portfolio’s highly
liquid Treasury securities as collateral in a repo transaction so as to obtain other funds to invest further in Treasuries or other securities. 63 64 BASICS OF FIXED INCOME SECURITIES Table 2.7 Discount Factors Z(0, T) on December 31, 1993 Maturity Coupon Bid Ask Z B (0, T) ZA (0, T) 19940630 19941231 19950630 19951231 19960630
19961231 19970630 19971231 19980630 19981231 0.000 7.625 4.125 4.250 7.875 6.125 6.375 6.000 5.125 5.125 98.3911 103.8125 100.1875 100.0000 108.6875 104.5313 105.1563 103.7813 100.0625 99.6250 98.4012 103.8750 100.2500 100.0625 108.7500 104.5938 105.2188 103.8438 100.1250 99.6875 0.9839 0.9639 0.9423 0.9191 0.9014
0.8743 0.8466 0.8203 0.7944 0.7703 0.9840 0.9645 0.9429 0.9196 0.9019 0.8748 0.8471 0.8208 0.7950 0.7708 Data Source: CRSP. Column (3) computes the balance of the financial institution. The corporation may purchase insurance against such scenarios, by purchasing a financial option, called a cap, that pays only if the floating reference
interest rate increases above some cutoff point, called the strike rate. Lower Sensitivity to Interest Rates: The higher the coupon rate, the larger are cash flows in the near future compared to the long-term future. The tails of the normal distribution are thin, in the sense that they give an extremely low probability to large events, which instead in
reality occur with some frequency. Several securities have embedded American options, including callable bonds and mortgage backed securities. As an example, the duration of zero coupon bonds is just their time to maturity. Fact 2.8 Consider a coupon bond at time t with coupon rate c, maturity T and payment dates T1 , T2 ,...,Tn = T . Additionally,
a 404 Not Found error was encountered while trying to use an ErrorDocument to handle the request. Fixed income securities present many risks for investors, even if they are issued or guaranteed by the U.S. government and therefore they are default free. When we compute the cash flow matrix: ⎞ ⎛ 1 c (T1) c1 (T2) ... Three-year, 15% fixed coupon
bond. For instance, the City of Chicago issued bonds for $983,310,000 in 2003 to pay for an expansion project of its O’Hare International Airport. 102 BASICS OF INTEREST RATE RISK MANAGEMENT Table 3.7 Two Term Structures of Interest Rates Maturity 02/15/94 Yield (c.c.) 02/15/94 Z(t, T) 05/13/94 Yield (c.c.) 05/13/94 Z(t, T) 0.25 0.50 0.75
1.00 1.25 1.50 1.75 2.00 2.25 2.50 2.75 3.00 3.25 3.50 3.75 4.00 4.25 4.50 4.75 5.00 5.25 5.50 5.75 6.00 3.53% 3.56% 3.77% 3.82% 3.97% 4.14% 4.23% 4.43% 4.53% 4.57% 4.71% 4.76% 4.89% 4.98% 5.07% 5.13% 5.18% 5.26% 5.31% 5.38% 5.42% 5.43% 5.49% 5.53% 0.9912 0.9824 0.9721 0.9625 0.9516 0.9398 0.9287 0.9151 0.9031 0.8921 0.8786
0.8670 0.8531 0.8400 0.8268 0.8145 0.8023 0.7893 0.7770 0.7641 0.7525 0.7418 0.7293 0.7176 4.13% 4.74% 5.07% 5.19% 5.49% 5.64% 5.89% 6.04% 6.13% 6.23% 6.31% 6.39% 6.42% 6.52% 6.61% 6.66% 6.71% 6.73% 6.77% 6.83% 6.86% 6.89% 6.93% 6.88% 0.9897 0.9766 0.9627 0.9495 0.9337 0.9189 0.9020 0.8862 0.8712 0.8558 0.8406 0.8255
0.8117 0.7959 0.7805 0.7663 0.7519 0.7387 0.7251 0.7106 0.6977 0.6846 0.6713 0.6619 Notes: Yields are calculated based on data from CRSP (Daily Treasuries). The balance sheet of such financial institution may look like the one in Table 3.5. Total assets are around $2.4 billion, with a dollar duration of $19.74 billion. Chapter 17 introduces the
second important result of continuous time finance, namely, the Feynman Kac formula, which provides the solution to the fundamental pricing equation obtained in Chapter 16. These spreadsheets and computer codes should be used as a guide not only to better understand the examples themselves, but also to carry out a similar analysis in the end-of-
chapter exercises. How do these rates move together? The problem is that we might not necessarily find all the data we want. 6 Interest Rate Derivatives: Futures and Options. In brief, VaR answers the following question: With 95% probability, what is the maximum portfolio loss that we can expect within a given horizon, such as a day, a week or a
month? • If the interest rate r 2 (0, 5) rises, the discount rate increases. The notion of duration then measures the impact that such a uniform change on the yield curve has on the price of the security P . The investor in Exercise 2 is still worried about interest rate volatility. Monthly Variation 3 2 1 0 −1 −2 −3 1955 1960 1965 1970 1975 1980 1985
1990 1995 Panel B. In fact: Fact 2.10 Two different bonds that have the same maturity but different coupon rates c have different yield to maturities y. How steep can the decline in spot rates be? exchange rate between having $105 in one year or $100 today, that is 0.9524 = $100/$105. I am also indebted with the copy editor, Peggy Monahan-

Pashall, who went through the 800 pages of the manuscript, and not only corrected all my English mistakes, but provided valuable constructive feedback on the write up itself. Compounding frequency: This is the frequency with which interest on an investment is accrues over time. To keep things simple, we assume that the inverse floater promises to pay 15% minus the short-term interest rate on an annual basis with 3 years maturity. Panel A of Figure 3.3 shows the monthly changes in the level of interest rates, while Panel B makes a histogram of these variations. Second, the book clarifies that models are "just models", and they are always an incomplete description of a much more complex real world. Course I: Introduction to Fixed Income Securities Parts I and II introduce basic analytical tools, and students familiar with basic calculus should be able to follow them relatively easily. 4. For instance, the old idea that an increasing yield curve predicts higher future interest rates has been proven false in the data time and again, and we should teach our students the implications of this empirical evidence. The difference in spreads provide the total gains that can be split between the two firms: Gains from trade = Fixed spread − Floating spread = 3% − 1% = 2% (1.2) In the example we divided the gains from trade equally between the two firms. Two famous examples come to mind, namely, the savings and loan crisis in the 1980s and the bankruptcy of Orange County, California, in 1994. This lower price is due to the lack of liquidity of bonds that have been issued long in the past compared to the latest issued T-notes used to compute the discount curve Z(0, T). This procedure can be iterated forward to obtain additional terms. |. This option has a value on its own. Chapter 13 illustrates a powerful methodology for valuing and hedging complicated securities, namely, the Monte Carlo simulations methodology. All rights reserved. If the trader does not have the bond to sell outright, then he or she can enter into a reverse repo with a repo dealer to obtain the bond to sell. As illustrated in the case, the Orange County portfolio could well have been mainly invested in short-term Treasuries and leveraged inverse floaters. To verify the yield to maturity computed are indeed correct, Table 2.2 also reports the discount factors Z y (0, T) = (1 + y/2)−2×T implied by Equations 2.33 and 2.34. 2.4.4 Quoting Conventions We end this section on Treasury bonds with a few remarks on the market quoting convention for Treasury bills and Treasury bonds. 3 Comparing the discount factor series 2 Data excerpted from CRSP (Fama Bliss discount bonds) ©2009 Center for Research in Security Prices (CRSP), The University of Chicago Booth School of Business. In this case, the continuously compounded interest rates are r(t, T1) = − ln(0.995399) = 1.8444%; 0.25 THE TERM STRUCTURE OF INTEREST RATES r(t, T2) = r(t, T3) = 39 ln(0.990142) = 1.9814%; 0.5 ln(0.978716) − = 2.1514%. Important in futures contracts is the fact that either counterparty may be called to make payments in the future. 2 The top solid line is the 3-months discount factor, the middle dotted line is the 1-year discount factor, and bottom dashed line is the 3-year discount factor. This is one equation in one unknown, and so we can solve for the Z(t, T 3) to obtain Z(t, T3) = = = $99.1093 − $1.5 × (Z(t, T1) + Z(t, T2)) $101.5 $99.1093 − $1.5 × (0.983607 + 0.965542) $101.5 0.947641 If we have a sufficient amount of data, we can proceed in this fashion for every maturity, and obtain all of the discount factors Z(t, T). This book provides a thorough discussion of these complex securities, the forces affecting their prices, their risks, and of the appropriate risk management practices. The Treasury issues these long term bonds every six months. How can we compute the gains from a swap trade? We apply this methodology to real world securities, such as corridor notes, ammortizing index swaps, mortgage backed securities, and collateralized debt obbligations. EXAMPLE 3.7 A portfolio manager has $100 million invested in a bond portfolio with duration DP = 5. While we can think of swaps as derivatives, in the more generic sense as hedging devices or non-funded financial instruments, their valuation does not derive from anything in particular, but only from the demand and supply of investors who use them for their needs to hedge or speculate in interest rates. More precisely, if we let n denote the number of compounding periods per year (e.g., n = 2 corresponds to semi-annual compounding), we obtain the following: Fact 2.6 Let the discount factor Z(t, T) be given, and let r n (t, T) denote the (annualized) n-times compounded interest rate. For portfolio B the investor would spend the same on all other securities except for the 7-year coupon bonds (paying 10% semiannually) from which the investor will short enough to get to $50 million. Derivative Pricing as Present Discounted Values of Future Cash Flows 9.3.1 Risk Premia in Interest Rate Securities 9.3.2 The Market Price of Interest Rate Risk 9.3.3 An Interest Rate Security Pricing Formula 9.3.4 What If We Do Not Know p? That is Wt+1 = Wt × (1 − xt) + Wt × xt × Capital gain on T-bond (3.41) + Interest in (9) at t + coupon in (10) (3.42) − Annuity coupon ($28,767) (3.43) Notice from the last row in Table 3.3 that the strategy still leaves $69,375 at maturity. Once again, a higher interest rate (across maturities) implies that short-term cash flows have a relatively higher weight in the value of the bond, and thus a lower sensitivity to changes in interest rates. The duration approximation in Equation 3.28 is appropriate for small parallel changes in the level of interest rates. As mentioned earlier, this terminology means that after six months the investment grows to $102.5 = $100 × (1 + 5%/2), which is then reinvested at the same rate for another six months, yielding at T the payoff: Payoff at T = $105.0625 = ($100) × (1 + r/2) × (1 + r/2) = ($100) × (1 + r/2)2 Given that the initial investment is $100, there are no cash flows to the investor during the period, and the payoff at T is risk free, the relation between money at t ($100) and money at T (= $105.0625 = payoff at T) establishes a discount factor between the two dates, given by Z(t, T) = $100 1 = payoff at T (1 + r/2)2 This example underlies the following more general statement: Fact 2.4 Let r2 (t, T) denote the (annualized) semi-annually compounded interest rate between t and T . In addition, for several fixed income securities the notion of yield to maturity is not well defined, because they may have floating rate coupons or embedded options. To keep the analysis simple, suppose the arbitrageur can enter into term repos with maturity of six months and that the haircut is zero. After the description of futures and options contracts, several examples discuss the usefulness of these contracts for risk management. Panel B of Figure 3.1 shows the historical observations of the level of interest rates up to 2005. First of all, the formal definition: Definition 1.2 A repurchase agreement (repo) is an agreement to sell some securities to another party and buy them back at a fixed date and for a fixed amount. I devote a chapter to discussing the Federal Reserve system, and the relation among interest rates, the real economy, and inflation. However, for many securities that do not have fixed payments, the equivalence is broken. c (Tn) we end up with N rows (N = number of bonds) and n < N columns (n = number of maturities). The straight dotted line in the Figure is the tangent to the curve P z (r, t, T) at the point r = 6%. That number is the 95% monthly VaR computed using the historical distribution approach. Options 6.2.1 Options as Insurance Contracts 6.2.2 Option Strategies 6.2.3 Put-Call Parity 6.2.4 Hedging with Futures or with Options? By Pietro Veronesi c 2010 JohnWiley & Sons, Inc. It is important for students to realize this fact early on, and it is this challenge that makes the study of fixed income markets so fascinating. For instance, look again at strategy 2. 49 COUPON BONDS Table 2.2 Term Structure on February 15, 2008 Coupon Maturity Time to Rate (%) Date Maturity 4.125 4.500 4.875 4.750 4.125 5.000 5.000 4.875 4.375 3.875 4.250 4.000 4.250 4.000 4.250 4.500 4.875 4.625 4.750 3.500 8/15/2008 2/15/2009 8/15/2009 2/15/2010 8/15/2010 2/15/2011 8/15/2011 2/15/2012 8/15/2012 2/15/2013 8/15/2013 2/15/2014 8/15/2014 2/15/2015 8/15/2015 2/15/2016 8/15/2016 2/15/2017 8/15/2017 2/15/2018 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0 8.5 9.0 9.5 10.0 Bid Ask Mid 100.9844 102.6094 104.4766 105.5078 105.0859 108.2344 109.0000 109.1719 107.3281 105.1406 106.8125 105.2344 106.3281 104.3750 105.4063 106.7188 109.0000 106.9375 107.8750 97.8750 101.0156 102.6406 104.5078 105.5391 105.1172 108.2656 109.0313 109.2031 107.3594 105.1719 106.8438 105.2656 106.3594 104.4063 105.4375 106.7500 109.0313 106.9688 107.9063 97.9063 101.0000 102.6250 104.4922 105.5234 105.1016 108.2500 109.0156 109.1875 107.3438 105.1563 106.8281 105.2500 106.3438 104.3906 105.4219 106.7344 109.0156 106.9531 107.8906 97.8906 Bootstrap Spot Discount Curve 98.9590 98.1892 97.3147 96.2441 95.0822 93.7612 92.2213 90.6046 88.7259 86.9809 85.0858 83.1241 81.1114 79.0613 76.8759 74.8256 72.6763 70.8392 69.1582 68.1581 Discount with Discount with y = 3.7548% y = 3.6603% 2.0930 1.8274 1.8147 1.9141 2.0172 2.1473 2.3137 2.4666 2.6582 2.7896 2.9365 3.0806 3.2207 3.3564 3.5064 3.6251 3.7548 3.8306 3.8818 3.8334 98.1572 96.3484 94.5729 92.8301 91.1194 89.4403 87.7920 86.1742 84.5862 83.0274 81.4974 79.9956 78.5214 77.0744 75.6541 74.2600 72.8915 71.5483 70.2298 68.9356 98.2027 96.4378 94.7045 93.0024 91.3309 89.6895 88.0775 86.4945 84.9400 83.4134 81.9142 80.4420 78.9962 77.5765 76.1822 74.8130 73.4684 72.1480 70.8513 69.5779 Data excerpted from CRSP (Daily Treasuries) ©2009 Center for Research in Security Prices (CRSP), The University of Chicago Booth School of Business. 100% in Cash 2000 2500 1500 Number of Events Number of Events 200,000 1000 500 2000 1500 1000 500 0 −1,000,000 0 4,000,000 0 −2,000,000 Dollars 0 6,000,000 Dollars 3.5 plot the results of an analysis similar to the immunization strategy in Figure 3.4, but in which the investment in long-term T-bond x t is kept fixed at 100% (Panel A), 70% (Panel B), 30% (Panel C), 0% (Panel D). Federal funds futures for various maturities; 9. Once again, swaps do not have fixed income. Figure 2.4 The Term Structure of Interest Rates on Three dates 8 7.5 7 6.5 Interest Rate (%) 6 5.5 5 4.5 4 3.5 01/31/1994 07/29/1994 3 01/31/1995 2.5 0 1 2 3 4 5 6 Time to Maturity 7 8 9 10 Data Source: Center for Research in Security Prices As we did for discount factors, we can plot various points of the term structure of interest rates over time. Descriptive material is from the case study ERISK: Orange County, downloadedfrom . Indeed, cashing nothing but interest income may not produce enough funds on which to survive. The first block of markets comprises the traditional fixed income markets, including U.S. government debt securities, municipal bonds, federal agency securities and the money market. I cover briefly the concepts in Chapters 1 to 6, which serve mainly to set the notation for the course. Note that the duration drops from 8.13 to 6.90. But first of all: What is an option? We are more explicit about the information in this column below. The cum-coupon value is principal times the semi-annual interest rate r2 (1.5)/2 determined six months earlier, at t = 1.5. We can compute the present value as of t = 1.5 of the total cash flow at time T = 2, resulting in the ex-coupon price still equal to 100. In this case, the 99% maximum loss can be computed from the properties of the normal distribution, resulting in VaR = −(μ P −2.326 ×σdP) = $598 million. The size of the U.S. debt market is no longer dominant in fixed income markets, as other markets became even larger, notably the mortgage backed securities market and the derivatives market. However, the trader, who wants to speculate on the decrease in the bond price, is happy to forgo part or all of the repo rate in order to get hold of the bond. 52 BASICS OF FIXED INCOME SECURITIES 2.5 FLOATING RATE BONDS Floating rate bonds are coupon bonds whose coupons are tied to some reference interest rate. Such errors can generate a large error in the VaR computation. Chapters 5 and 6 cover popular fixed income derivatives, such as forward rates, swaps, futures, and options, and their uses by market participants. Figure 2.6 compares three methodologies of computing the term structure of interest rates: The bootstrap, the Nelson Siegel model, and the Extended Nelson Siegel model, further discussed below. 3.3.1 Cash Flow Matching and Immunization Ms. Caselli, in Example 3.9, can purchase an annuity from a financial institution. 88 BASICS OF INTEREST RATE RISK MANAGEMENT Figure 3.3 Changes in the Level of Interest Rates: 1965 - 2005 Interest Rate (%) A: Monthly Changes in the Level of Interest Rates 4 2 0 −2 −4 1965 1970 1975 1980 1985 1990 1995 2000 2005 2 3 4 10 15 20 B: Histogram of Monthly Changes in the Level of Interest Rates Occurrences 40 30 20 10 Probability Density 0 −4 −3 −2 −15 −10 −1 0 1 Interest Rate (%) C: Probability Distribution of Portfolio P/L 0.4 0.3 0.2 0.1 0 −20 −5 0 Millions of Dollar 5 Data Source: CRSP. Similarly to the mortgage backed securities market, the asset backed securities market involves the issuance of debt instruments to investors, collateralized by various types of loans, such as auto loans, credit cards, and the like. 12 To understand the logic, consider first the following simple example EXAMPLE 2.13 10 In this chapter we only review the pricing of floating rate bonds for the case in which the coupon rate is linked to the same interest rate that is also used for discounting purposes, which greaty simplifies the analysis and provides the formulas needed for future applications. From the answers to the Exercises 7 - 11, answer the following: (a) What is the change in value in the portfolio due to the change in time only, without change in interest rates? For instance, the leveraged inverse floater discussed in Section 2.8.4 of Chapter 2 has a coupon c(t) = 25% − 2 × r1 (t − 1) (3.51) CASE STUDY: THE 1994 BANKRUPTCY OF ORANGE COUNTY 107 Table 3.9 The Duration of the Leverage Inverse Floater Security Value Weight w Duration D D∗w 2 × Pz (3) Pc(3) −2 × PF R (3) 174.91 156.41 -200.00 1.3320 1.1911 -1.5231 3.00 2.5448 1.00 3.9959 3.0311 -1.5231 Total Value: 103.78 Duration: 5.5040 Recall also from Chapter 2 that the price of the leveraged inverse floater can be computed as:7 Price leveraged inverse floaterPLIF (0, T) = 2 × Pz (0, T) + Pc(0, T) − 2 × PF R (0, T) (3.52) To compute the duration of the leveraged inverse floater we need to compute the duration of the fixed-coupon bond. The last two columns of the table provide the discount factors Z(0, T) computed from either bid prices or ask prices.16 2.8.3 Calculating the Price of the Inverse Floater Recall that the value of the inverse floater can be computed from the value of a zero coupon bond, a coupon bond with coupon rate equal to 15%, and a floating rate bond, all of them with maturity equal to three years. At the end of the chapter we also show how we can use this information to obtain the price of some simple structured securities, such as inverse floaters, which are popular securities if an investor wishes to bet on a decrease in interest rates. 1.6.1 Swaps Interest rate swap contracts were introduced in the early 1980s to take advantage of some apparent arbitrage opportunity that was surfacing in the corporate bond market. Let t = 0, for convenience, so that T denotes both maturity date and time to maturity. Why do they work? The LIBOR rate, the rate at which banks in London borrow from each other on an uncollateralized basis, is the main reference rate in numerous derivative securities. Duration of a portfolio of securities: This can be computed as the weighted average of durations of the individual securities in the portfolio, where the weights equal the percentage holdings of the securities. Coupon notes and bonds present an additional complication. 12. If at the end of the repo term the trader were to default, the repo dealer could sell the security and be made whole. Then the semi-annually compounded interest rate r 2 (t, T) can be computed from the formula 1 r2 (t, T) = 2 × −1 (2.4) 1 Z (t, T) 2×(T −t) 2.2.1.2 More Frequent Compounding Market participants' time value of money – the discount factor Z(0, T) – can be exploited to determine the interest rates with any compounding frequency, as well as the relation that must exist between any two interest rates which differ in compounding frequency. w1 = EXAMPLE 3.3 A bond portfolio manager has $100 million invested in 5-year STRIPS and $200 million invested in 10-year STRIPS. Foreign exchange rates; INTRODUCTION 7 11. 104 BASICS OF INTEREST RATE RISK MANAGEMENT Figure 3.6 The Level of Interest Rates, 1992 - 1994 7 6.5 Interest Rate (%) 6 5.5 5 4.5 4 3.5 1992 1992.5 1993 1993.5 1994 1994.5 1995 Data Source: CRSP. We do not delve any more into this issue, as it is beyond the scope of this chapter, but relevant readings are available in the references to this chapter at the end of the book. Let there be n coupon bonds, with coupons c i , maturities T i and prices denoted by P (t, Ti). More generically, a deep understanding of the forces that affect the valuation, risk, and return of fixed income securities and their derivatives has never been so important. Treasury bills are issued very frequently, typically every week for bills up to six months, and every four weeks for one-year bills. Let Ti denote the coupon payments times, for i = 1, ..., n. Then, at any time t = .5, 1, 1.5, ..., 30 the financial institution: • collects the 4% 2 coupon from the 30-year bond; • collects the interest cumulated over the six months on the cash deposit; • pays the annuity cash flow of $28,767 to Ms. Caselli; and • reinvests the remaining balance in long-term bonds and overnight deposit according to the rule: Duration of annuity Percentage investment in long-term bond = x t = Duration of long-term bond. While it is still hard to forecast how long the recession will last, a certain fact for now is that fixed income markets will get bigger. Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. Three-year floating rate bond. 62 BASICS OF FIXED INCOME SECURITIES in Chapter 1) the price of an inverse floater is then: Price inverse floater = Pz (0, 3) + Pc (0, 3) − PF R (0, 3) (2.43) where we recall that Pz (0, 3), Pc (0, 3), and PF R (0, 3) denote the prices of a zero coupon bond, a coupon bond, and a floating rate bond with three years to maturity. 1.1.2 No Arbitrage and the Law of One Price At the source of the ripple-through effect from one market to the next is the notion of no arbitrage. The continuously compounded interest rate is obtained by increasing the compounding frequency n to infinity. Table 1.6 shows the average daily amount outstanding in these contracts. c1 (Tn) c2 (Tn) .. In this case, we find it equal to $715 million. Then Equation 3.28 implies that dP has a normal distribution with mean and standard deviation given by: (3.29) μP = −DP × P × μ and σ P = DP × P × σ. These are securities that pay a sequence of coupons and the principal back at maturity. If we restrict the sample to compute the standard deviation of interest rate changes dr to the more recent period, such as five years, we find the much smaller standard deviation σ dr = 0.0028. These notes carry a fixed coupon that is paid semi-annually up to the maturity of the note. 26 AN INTRODUCTION TO FIXED INCOME MARKETS Intuitively, an option is the financial equivalent of an insurance contract: It is a contract according to which the option buyer, who purchases the insurance, receives a payment from the option's seller, who sold the insurance, only if some interest rate scenario occurs in the future. That is, the world is much more complicated than the simple models or formulas would imply. However, the buyer agrees to pay the seller any accrued interest between the last coupon date and purchase price. Given this information, we can now compute the duration of the leveraged inverse floater. This part should therefore be useful to link this material to the notion of implied volatility from the Black formula, the standard market formula used to quote standard derivatives. 2.2 INTEREST RATES Grasping the concept of a rate of interest is both easier and more complicated than absorbing the concept of a discount factor. Do you obtain the same price? Today, however, the U.S. government debt is no longer the dominant fixed income market, not so much because the U.S. debt shrank over the past two decades, but rather because other fixed income markets rose substantially relative to U.S. debt and became the main reference for fixed income pricing. 40 BASICS OF FIXED INCOME SECURITIES Figure 2.3 The Shapes of the Term Structure B: 11/30/2000 : Decreasing 6.4 7 6.2 Interest Rate (%) Interest Rate (%) A: 10/30/1992 : Increasing 8 6 5 4 5.8 5.6 3 2 6 0 2 4 6 Time to Maturity 8 5.4 10 0 4 6 Time to Maturity 8 10 D: 07/31/1989 : Inverted Hump 6.4 8 6.3 7.9 Interest Rate (%) Interest Rate (%) C: 03/31/2000 : Hump 2 6.2 6.1 6 5.9 7.8 7.7 7.6 7.5 0 2 4 6 Time to Maturity 8 10 7.4 0 2 4 6 Time to Maturity 8 10 Data Source: Center for Research in Security Prices structure that is first decreasing and then increasing. Recall also that the repo rate is decided at time t. Note that in this example, there is no difference between the daily compounded interest rate (n = 252) and the one obtained with higher frequency (n > 252). That is, only a long position in the fixed rate and short position in the floating rate bond does not exactly mimic an inverse floater. CHAPTER 2 BASICS OF FIXED INCOME SECURITIES 2.1 DISCOUNT FACTORS Receiving a dollar today is not the same as receiving it in a month or in a year. Column (8) reports the fraction of capital x t invested in the 30-year bond, obtained from using Equation 3.38. What happens if the trader does not return the security? We apply such a strategy for instance in Chapter 8 in the context of mortgage backed securities. This definition of duration is simple to apply in order to compute the duration of interest rate securities, from zero coupon bonds to portfolios of securities. The economic expansion that started in 1991, which would end in 2001, also generated a government surplus between 1996 and 1999, which led the U.S. government to initiate a policy of debt buyback. Second, for longer maturities not all of the bonds may be available. And finally, the whole fixed income derivatives market, which includes forwards, futures and options, adds a few more trillion dollars. 37 INTEREST RATES Table 2.1 Interest Rate and Compounding Frequency n r n (t, t + 1) Annual Semi-annual Monthly Bi-monthly Weekly Bi-weekly Daily Bi-daily Hourly 1 2 12 24 52 104 365 730 8760 5.000% 4.939% 4.889% 4.883% 4.881% 4.880% 4.879% 4.879% 4.879% Continuous ∞ 4.879% Compounding Frequency EXAMPLE 2.5 Consider the earlier example in which at t we invest $100 to receive $105 one year later. Published by John Wiley & Sons, Inc., Hoboken, New Jersey. Let x t % denote the fraction of the total capital – $1,000,000 at initiation – invested in the 4%, 30-year bond, as described in Example 3.9. Assume that the remaining (1 − xt)% is kept as cash in a deposit account, thereby yielding the overnight deposit rate. Fleming and Kenneth D. Therefore, the procedure is instead to find parameters (θ 0 , θ1 , θ2 , λ) that minimize the quantity J(θ 0 , θ1 , θ 2 , λ) in Equation 2.55. DISCOUNT FACTORS 31 This example highlights an important property of discount factors. Thus, overall, we have Price leveraged inverse floater = 2 × Pz (0, 3) + Pc (0, 3) − 2 × PF R (0, 3) (2.48) We already know from the previous section the prices P z (0, 3) = $87.45 and PF R(0, 3) = $100. Chapter 7 discusses the Federal Reserve policy rules, and covers in particular the Federal Funds rate. However, if the interest accrues every half a year, for instance, the correct answer is given by r = 4.939%. The changes in these markets are evident also in Figures 1.1 and 1.2. Considering first the Treasury debt market, we see that from 1986 to 1996 it grew steadily. That bond is said to be "on speacial." The profit from the reverse repo transaction is then Profit = (Pt − PT) + Repo interest where the repo interest is computed as in Equation 1.1, namely, the amount deposited with the repo dealer (P t) times the repo rate times n/360, where n is the number of days between the two trading dates t and T . The hands-on approach will make clear why practitioners use one model or another in the various circumstances: Students will experience firsthand the difficulties of dealing with data even when using relatively simple models. As can be seen, they both go up and down 6 Data excerpted from CRSP (Fama Bliss Discount Bonds and Fama Risk Free Rate) ©2009 Center for Research in Security Prices (CRSP), The University of Chicago Booth School of Business. The U.S. Treasury does not issue these securities directly to investors, but investors can purchase them and hold them through financial institutions and government securities brokers and dealers. 2.9.3 Curve Fitting Let's consider approaching the problem from a completely different angle. Still, as discussed below, Parts I and II are sufficient to cover a complete course in fixed income, and they do cover all of the deep concepts that I believe anyone who studies fixed income and plays any role in these markets should possess. How can the financial institution now hedge this commitment to pay exactly $28,767 twice a year for 30 years? In contrast, a short position means that the investor sold the bond without actually having it in the portfolio. The shape of the term structure of interest rates is not always increasing. See Chapter 6. Treasury bills (T-bills) are short-term debt instruments, with maturity up to one year.2 They do not pay any cash flow over time, only the principal at maturity. But this is in fact exactly the reason for a new measure of risk: The expected shortfall is very useful precisely for those situations in which the portfolio losses are not expected to be normally distributed. This price implies a discount factor on that date equal to Z(t 1 , T1) = 0.99115. 3 Since borrowing is collateralized by the value of the asset, the repo rate is lower than other borrowing rates available to banks, such as LIBOR. Denoting L 1 , L2 ,...,Lm the current value of each of its m liabilities (excluding equity), and DL,1 , DL,2 ,...,DL,m their durations, we obtain Duration of liabilities D L = n wL,i DL,i i=1 where Li wL,i = n i=1 Li The aim of asset - liability management is often taken to minimize the impact that the variation in the level of interest rates has on the value of equity. The chapter also discusses a simple methodology to build long-term trees from the prediction of future short-term interest rates, as well as the concept of risk adjusted probabilities and risk premia. The problem with polynomial functions is that they do not allow for a sufficient number of shapes, without APPENDIX: EXTRACTING THE DISCOUNT FACTORS Z(0, T) FROM COUPON BONDS 71 going into a very high order polynomial. There are three main ways: 1. How do you know that changes in value are not a product of coupon payments made over the period? Consider a portfolio made up of N 1 units of security 1, and N2 units of security 2. Fact 3.2 Let Pz (r, t, T) be the price of a zero coupon bond at time t with maturity T and continuously compounded interest rate r. Because the borrowing is collateralized, the rate is lower than the LIBOR rate, for instance. . This difference is called the term spread, or slope, of the term structure of interest rates. 3 The duration definition in Equation 3.1 is often referred to as the "modified duration," to differentiate it from the Macaulay definition of duration, discussed below. Consider an arbitrageur who is contemplating going long a 4% coupon bond by borrowing at the current floating rate. The outcome of the two procedures is almost the same. As can be seen, large increases and decreases are not very likely, but they do occur occasionally. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, (978) 750-8400, fax (978) 646-8600, or on the web at www.copyright.com. 3.6 EXERCISES 1. It appears that the portfolio had about $2.8 billion in "inverse floaters [...], index amortizing notes, and collateralized mortgage obbligations." 8 For simplicity, we assume that $2.8 billion was invested only in leveraged inverse floaters. 5 See Bloomberg.com Web site . The 6-month, 99% VaR in this case is $668 million, a large number, but much smaller than the actual ex-post losses. 13 The spread s often reflects a lower credit quality than the reference rate used. Part II: Binomial Trees The second part of the book introduces readers to the concept of term structure modeling and no arbitrage strategies. 2 The substantial loss from the total asset pool of $7.5 billion forced Orange County to declare bankruptcy. Then r n (t, T) is defined by the equation Z(t, T) = 1+ 1 rn (t,T) n n×(T −t) (2.5) Solving for r n (t, T), we obtain rn (t, T) = n × 1 1 Z (t, T) n×(T −t) −1 (2.6) For instance, a $100 investment at the monthly compounded interest rate r 12 (0, 1) = 5% yields by definition r12 (0, 1) r12 (0, 1) ×· · · [12 times] · · ·× 1 + = $105.1162 Payoff at T = $100× 1 + 12 12 Thus, the monthly compounded interest rate r 12 (0, 1) = 5% corresponds to the discount factor Z(0, 1) = $100/$105.1162 = 0.95133, and vice versa. PREFACE xxi About this Book This book covers "fixed" income securities, their valuation, their risks, and the practice of risk management. Figure 2.3 provides examples of spot curves r(t, T) at four different dates. In the fifth column we compute the discounted cash flows, the sum of which give the price P c (0, T) = $107.795 at the bottom of the table. Pietro Veronesi Chicago June, 2009 ACKNOWLEDGMENTS First of all, I want to thank my students at the University of Chicago Booth School of Business, whose enthusiasm for fixed income securities convinced me to write this book. However, when the size of a derivative market becomes larger than the one of the primitive securities, which price depends on which is not clear at all. Unless otherwise specified, we therefore define a floating rate bond as follows: Definition 2.7 A semi-annual floating rate bond with maturity T is a bond whose coupon payments at dates t = 0.5, 1, ..., T are determined by the formula Coupon payment at t = c(t) = 100 × (r 2 (t − 0.5) + s) (2.38) where r2 (t) is the 6-month Treasury rate at t, and s is a spread. The data analysis is an integral part of the book and the learning experience. That is, Interest payment [column (9)] = W t × (1 − xt) × rt /2 (3.39) Similarly, Column (10) represents the total coupon received from the 30-year bond investment Coupon payment [column (10)] = Wt × xt × 4%/2 Price T-bond in Column (6) (3.40) Finally, returning to Column (3), the total amount of capital at the institution is updated by taking into account inflows and outflows. To perform the bootstrap methodology, we then must cherry pick the bonds that we deem have the highest liquidity (e.g., notes over bonds). Thus, the duration of the portfolio is Duration of portfolio = 200 100 ×5+ × 10 = 8.3 300 300 Therefore, a one basis point increase in interest rates generates a portfolio loss of Loss in portfolio value = $300 million × 8.3 × 0.01% = $249, 000 Generalizing the formula in Equation 3.8 to n securities, we obtain: Fact 3.3 The duration of portfolio of n securities is given by DW = n wi Di (3.10) i=1 where wi is the fraction of the portfolio invested in security i, and D i is the duration of security i. However, once again, the correct answer really depends on the frequency with which interest accrues on the security. The example in the Panel D is for July 31, 1989. For given parameter values (θ0 , θ1 , θ2 , λ), it is possible to compute the value of bond prices implied by the Nelson Siegel model. 90-day Eurodollar futures for various maturities; 8. But options contracts are a vital part of the fixed income market. Firm B, instead, can borrow fixed at 12% or floating at LIBOR + 2%. For instance, in Chapters 3 and 4 we discuss the use of repurchase agreements to increase portfolio leverage, in Chapter 5 we illustrate the use of the repo market to carry out a swap spread arbitrage trade, and in Chapter 16 we use 3 The amount outstanding of repurchase agreements need not equal the amount outstanding of reverse repurchase agreements, as each column reflects the size of collateralized borrowing or lending of security dealers only, and not the whole universe of repo counterparties. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008. It is important to realize, though, that the VaR so computed is a monthly figure, while Orange County losses accrued over a six-month period. This chapter also links back to Chapter 11 in what concerns the notion of implied volatility. Why do discount factors vary over time? Looking at interest rates from a different perspective, consider an investment in a security that costs $100 today and that pays $105 in one year, as in our earlier example. That is, DL ≈ 0. Chapter 16 discusses the notion of dynamic hedging, that is, the practice by market participants of frequently rebalancing their portfolios to hedge their risk exposure. Special Repo Rate: At times, one particular Treasury security is in high demand and hence the repo rate on that security falls to a level substantially below the GCR. The numbers we computed vary greatly depending on (a) the type of model (e.g., normal versus historical); (b) the horizon (one month versus six months); (c) the sample used (last five years versus longer sample); (d) the type of risk measure (VaR versus Expected Shortfall). 1 dPz (r, t, T) Dz,T = − (3.5) Pz (r, t, T) dr 1 = − × [−(T − t) × Pz (r, t, T)] Pz (r, t, T) = T −t (3.6) The duration of a zero coupon bond is given by its time to maturity T − t. 44 BASICS OF FIXED INCOME SECURITIES Therefore, the price of the note on that date was Pc(t, Tn) = $2.1875 × 4 Z(t, t + 0.5 × i) + $100 × 0.91707 = $99.997, i=1 which was indeed the issue price at t. 9 It should be mentioned that by this date Value-at-Risk and expected shortfall were not yet been introduced as risk measures, and therefore Orange County's Treasurer could have not done the following calculations. If the law of one price does not hold for some securities, then an arbitrage opportunity exists. Given an interest rate and its compounding frequency, we can define a discount factor. In particular, the trader can enter into a reverse repo with another repo dealer at a negative repo rate to obtain the Treasury security to deliver to the original counterparty. This latter number, which is equivalent to the 5% rate of interest, perhaps less intuitively describes the return on an investment. Investment horizons affect the interest rate to be received on an investment or paid on a loan. Therefore, we have the equality: Price floating rate bond with spread s = Price floating rate bond with zero spread n Z(0, t) +s × t=0.5 Rate r2 (2) r2 (1.5) r2 (1) r2 (0.5) r2 (0) Time 2 1.5 1 0.5 0 - c (0.5) = 100 × c (1) = 100 × r2 (0) 2 r2 (0.5) 2 r2 (1) 2 r2 (1.5) 2 c (1.5) = 100 × c (2) = 100 × Coupon PF R (0) r (0) 1+ 22 r (0) 100× 1+ 22 r (0) 1+ 22 C (0.5) PF R = 100 = = r (0.5) 1+ 2 2 C (1) PF R r (0.5) 1+ 2 2 r (0.5) 100× 1+ 2 2 = 100 = r (1) 1+ 22 r (1) 100× 1+ 22 r (1) 1+ 22 r (1.5) 1+ 2 2 C PF R (1.5) = 100 = C (2) PF R r (1.5) 1+ 2 2 r (1.5) 100× 1+ 2 2 = 100 = PF R (0.5) PF R (1) = PF R (1.5) = PF R (2) = 100 Ex-Coupon Price r2 (1.5) 2 PFCR (0.5) = PF R (0.5) + c (0.5) = 100 × 1 + r22(0) PFCR (1) = PF R (1) + c (1) = 100 × 1 + r2 (0.5) 2 PFCR (1.5) = PF R (1.5) + c (1.5) = 100 × 1 + r22(1) c (2) PFCR (2) = 100 + = 100 × 1 + Cum-Coupon Price Table 2.3 The Valuation of a 2-year Floating Rate Bond FLOATING RATE BONDS 55 56 BASICS OF FIXED INCOME SECURITIES At reset dates the price of the floating rate bond with zero spread is just par (=100), so that 13 Price floating rate bond with spread s = 100 + s × T Z(0, t) (2.39) t=0.5 The second complication is that the valuation may be outside reset dates. How is the term spread determined? Consider an investment of $100 at t at the semi-annually compounded interest r = 5%, for one year. The U.S. debt started growing again in 2001, to reach about $5.9 trillion in December 2008. More specifically, borrowing at the 6-month term repo is essentially equivalent to shorting a 6-month floating rate bond. Because we are interested in understanding the dynamics behind Orange County's financial problems, we assume that we are beginning our analysis on December 31, 1993 (a year before the county declared bankruptcy). Example 2.1 illustrates this point. We discuss methodologies to estimate discount factors from bond data in Section 2.4.2 and in the Appendix. Let Pt denote the (invoice) price of the bond at time t. xxx PREFACE Course II: An Advanced Course in Fixed Income Securities The advanced course would make full use of Part II and Part III of the book. That is, all of these instruments are highly correlated. PiVe Investment Group and the Hedging of Pass-Through Securities 8.7.1 Three Measures of Duration and Convexity 8.7.2 PSA-Adjusted Effective Duration and Convexity 8.7.3 Empirical Estimate of Duration and Convexity 8.7.4 The Hedge Ratio Appendix: Effective Convexity 324 325 325 326 328 330 CONTENTS xi PART II TERM STRUCTURE MODELS: TREES 9 ONE STEP BINOMIAL TREES 335 9.1 335 338 338 338 340 343 344 344 345 346 347 348 349 349 350 351 352 353 9.2 9.3 9.4 9.5 9.6 10 MULTI-STEP BINOMIAL TREES 357 10.1 10.2 357 358 359 361 365 365 367 369 372 376 376 10.3 10.4 10.5 10.6 10.7 11 A one-step interest rate binomial tree 9.1.1 Continuous Compounding 9.1.2 The Binomial Tree for a Two-Period Zero Coupon Bond No Arbitrage on a Binomial Tree 9.2.1 The Replicating Portfolio Via No Arbitrage 9.2.2 Where Is the Probability p? 2.1.1 Discount Factors across Maturities Definition 2.1 and Example 2.1 highlightthat the discount factor at some date t (e.g., August 10, 2006) depends on its maturity T (e.g., February 8, 2007). It is market convention to quote Treasury notes and bonds without any inclusion of accrued interests. The total size of these debt markets is around $15 trillion. In turn, this expectation as well as the rules of no arbitrage, discussed below, have an impact on other short-term borrowing rates, such as the short-term LIBOR, the short-term Eurodollar rates and so on. The financial institution can engage in a dynamic immunization strategy. My homeworks are always based on real-world securities that need to be priced, hedged or, more generically, analyzed, and I wrote most of the end-of-chapter exercises in this book with this aim in mind, namely, to have students analyze real-world securities by using the models discussed in each chapter. This relation forms the basis of much of the analysis that follows in later chapters, and so it is particularly important. It changes the future cash flow c(1) = 100 × r2 (0.5) 54 BASICS OF FIXED INCOME SECURITIES • If the interest rate r 2 (0.5) rises, the future cash flow increases. The next sections illustrates additional methodologies used in practice to deal with such large quantities of data. As discussed in Section 2.4, a coupon bond with coupon rate c and n future coupon payments can be considered a portfolio of zero coupon bonds, in which c/2 is invested in the first n − 1 zeros, and 1 + c/2 in the n−th zero: Pc (0, Tn) = n−1 i=1 c c Pz (0, Tn) × Pz (0, Ti) + 1 + 2 2 (3.11) 80 BASICS OF INTEREST RATE RISK MANAGEMENT The duration of a coupon bond can then be computed by using Equation 3.10. 8 Basics of Residential Mortgage Backed Securities. We assume that both dates correspond, approximately, to T 1 . In addition, the portfolio contained some leveraged inverse floaters.17 The main difference between these and the plain vanilla inverse 17 See Mark Grinblatt and Sheridan Titman, Financial Markets and Corporate Strategy (2nd Edition), McGraw-Hill Primis, 2006, Chapter 23. In particular, it covers some popular models for the pricing of fixed income instruments, as well as their estimation using real data. How do we compute the expected shortfall in this case? Typo List: Typos Solution Manual for Students : Student Solution Manual Excel file with data for exercises Book Home Page at Wiley : Fixed Income Securities: Valuation, Risk, and Risk Management Additional Material for Instructors : Wiley Instructors' Companion Site. Then, after we have understood the concept of no arbitrage, we can look back and try to understand why sometimes apparent arbitrage opportunities seem to appear in the market, in the form of spreads between securities that look similar. That is, there is only 1% probability that Orange County portfolio could lose more than $715 million in one month. 4 Given this distribution, we can compute the maximum loss that can occur with 95% probability. We will return to this issue in later chapters. Table 1.3 lists the types of securities. Instead of assuming that the change took place 6 months later, assume that the change in the yield curve occurred an instant after February 15, 1994. Thus, we can compute the duration of the inverse floater by applying the formula for the duration of a portfolio, namely, Equation 3.10. From basic Ordinary Least Squares (OLS) formulas, we then find −1 Z (0) = (C × C) C × P (0) For this procedure to work, however, we must have more bonds than maturities, which does not occur for longer maturities. 2.8.1 Decomposing Inverse Floaters into a Portfolio of Basic Securities An inverse floater is a security that pays a lower coupon as interest rates go up (hence the name inverse floater). Published simultaneously in Canada. Since −z xf(x)dx −f(−z) E[X|X < −z] = ∞−z = N (−z) ∞ f(x)dx the formula in Equation 3.34 follows from substituting this latter expression into Equation 3.55. This site contains Excel spreadsheets for selected chapters (updated versions also available here), Powerpoint slides for all chapters, the solution's manual for all chapters (and related spreadsheets / codes), and a test bank . Chapter 11 applies the methodology illustrated in the two earlier chapters to real-world securities. The mortgage is a fixed income instrument: It is a promise from the homeowner to make certain cash payments in the future. Just follow the research link. This Treasury bill would not make any other payment between the two dates. Chapter 7 describes this market in more detail. Summary Exercises Case Study. More specifically, a government zero coupon bond at time t with maturity T has a price equal to Pz (t, T) = 100 × Z(t, T) The subscript "z" is a mnemonic term for "zero" in zero coupon bond. Denoting t = June 5, 2008, and T1 , T2 , and T3 the three maturity dates, the implied discount factors are Z(t, T 1) = 0.995399, Z(t, T2) = 0.990142, and Z(t, T3) = 0.978716. Then, we can write Pc (0, T2) = = = = = c/2 × 100 100 × (1 + c/2) + 2 1 + r2 /2 (1 + r2 /2) c/2 × 100 100 1 + c/2 + × 1 + r2 /2 1 + r2 /2 1 + r2 /2 c/2 × 100 100 + ×1 1 + r2 /2 1 + r2 /2 100 × (1 + c/2) 1 + r2 /2 100 (2.17) (2.18) (2.19) (2.20) (2.21) This argument can be extended to many periods. Notice also that Equation 2.42 contains a further simplification, namely, the fact that the coupon is always positive, which would be violated if the short rate were ever larger than 15%. The world of fixed income securities has become more complex, and students who aim at working in this environment must now be able to recognise and work with this complexity. For instance, in Example 3.7 the 99% VaR is $5.52 million, which is higher than the figure obtained under the normal distribution approach ($4.83 million), but not much higher. The most immediate fact that springs out from Panel A of this figure is that all yields move up and down roughly together. As can be seen, the term structure of interest rates on October 30, 1992 was increasing, which is a typical pattern in United States. It is worth pointing out that in contrast the VaR measure does not capture well the risk embedded in the tails of the distribution. The U.S. government issues various types of securities. Unfortunately, this is hard to tell. This formula is at the basis of the risk neutral pricing methodology widely used by market participants to price fixed income securities. For instance, Figure 2.4 plots the term structure of interest rates on three different dates, at six-months intervals, namely, from January 31, 1994 to January 31, 1995. 5. The following provides a simple example: EXAMPLE 3.5 DURATION 81 Table 3.1 Duration of Coupon Bond, Coupon = 6% Period i Payment Time Ti Cash Flow CF Discount Z(0, Ti) Discounted Cash Flow CF ×Z(0, T0) Weight wi Weight ×T i wi × Ti 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0 8.5 9.0 9.5 10.0 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 103 0.976 0.952 0.929 0.906 0.884 0.862 0.841 0.821 0.801 0.781 0.762 0.744 0.725 0.708 0.690 0.674 0.657 0.641 0.626 0.610 2.927 2.855 2.786 2.718 2.652 2.587 2.524 2.462 2.402 2.344 2.286 2.231 2.176 2.123 2.071 2.021 1.972 1.923 1.877 62.858 0.027 0.026 0.026 0.025 0.025 0.024 0.023 0.023 0.022 0.022 0.021 0.021 0.021 0.020 0.020 0.019 0.019 0.018 0.018 0.017 0.583 0.014 0.026 0.039 0.050 0.061 0.072 0.082 0.091 0.100 0.109 0.117 0.124 0.131 0.138 0.144 0.150 0.155 0.161 0.165 5.831 Price 107.795 Duration 7.762 82 BASICS OF INTEREST RATE RISK MANAGEMENT Recall that in Section 2.5 of Chapter 2 we considered the price of a floating rate bond. And this for several reasons: First, governments' debt will expand in the future, as governments across the globe increase their spending to stimulate demand and jump start their economies. One popular model is the following: 2.9.3.1 The Nelson Siegel Model The Nelson Siegel model is perhaps the most famous model. As explained in Chapter 5, interest rate swaps, the single largest derivatives market (see Table 1.1), use LIBOR rates as the reference rates to determine the size of cash flows implied by a contract. This famous case highlights yet another example of the large losses that interest rate movements may bring about in portfolios that are "too sensitive" to interest rates. This implies that when interest rates go up, the price of inverse floaters receive a negative shock from two channels: 1. 1.7 ROADMAP OF FUTURE CHAPTERS In this chapter we described some of the major fixed income markets. For instance, in Table 2.6 there are 164 bonds with maturity of less than five years, but there are only 60 months in five years, implying that many months have multiple bonds maturing in them. For instance, on August 10, 2006 the Treasury issued a 182-day bill at a price of $97.477 for $100 of face value. These data are collected in spreadsheets, which are available with the textbook. Banks with a reserve surplus may then lend some of their reserves to banks with a reserve deficit. We could use any other compounding frequency, but as mentioned earlier, the continuously compounded frequency has some analytical advantages, as we shall see. However, about 10% of the U.S. debt is in Treasury Inflation Protected Securities (TIPS), that pay a coupon that is not fixed at all, but fluctuates together with the realized U.S. inflation rate. Given the discount factors in Table 2.7, we can obtain values for these standard bonds. This security is very sensitive to changes in interest rates, indeed. After introducing the main fixed income markets in Chapter 1, Chapter 2 contains the building blocks of fixed income relations, namely, the notion of discounts, interest rates, and the term structure of interest rates. The backward induction argument up to t = 0.5 still holds: At time t = 0.5 the ex-coupon bond price will be worth 100 and the cum-coupon bond price will be worth $101 = $100 + $100 × 2%/2. To see this, let us compute the total payoff at maturity T assuming that the investor can reinvest all of the coupons paid at dates T 1 , T2 , ..., Tn−1 at the constant rate y for the remaining periods T − T1 , T − T2 , ..., T − Tn−1 . When the horizon is longer, the discrepancy between the annualized interest rate figure and the annualized rate of return on the investment is larger. It is important to note that the duration is higher than the inverse floater maturity (3 years). To some extent, then, the yield to maturity is just a convenient way of quoting a bond price to other traders. While future chapters discuss the pricing, hedging and the risk involved in swaps, it is informative at this point to see the economic need that led to the creation of this market at the beginning of the 1980s. We can also see that the term spread, the distance between the two rates, changes over time as well. The solid line in Figure 2.6 plots the fitted yield curve according to the Nelson Siegel model. The value of the portfolio is then W = N1 × P1 + N2 × P2 Let D1 and D2 be the duration of security 1 and security 2, respectively. We can apply the definition of duration in Definition 3.2 and reorganize the expressions: 1 dW W dr d(N1 × P1 + N2 × P2) dr dP1 dP2 N1 × × + N2 × dr dr Duration of portfolio = D W = − 1 W 1 = − W = − (3.7) 79 DURATION = = = 1 1 dP1 1 dP2 N1 × P1 × − + N2 × P2 × − W P1 d r P2 d r N2 × P2 N1 × P1 D1 + D2 W W w1 D1 + w2 D2 (3.8) where N1 × P1 N2 × P2 and w2 = (3.9) W W The expression in Equation 3.8 shows that the duration of a portfolio is a weighted average of the durations of the assets, where the weights correspond to the percentage of the portfolio invested in the given security. The duration of the inverse floater can be computed then as DInverse = wZero × Dzero + wF ixed × DF ixed + wF loating × DF loating (3.50) where wZero = Pz (0, 3)/PIF (0, 3) = 0.7521, wF ixed = Pc (0, 3)/PIF (0, 3) = 1.1079, and wF loating = −PF R (0, 3)/PIF (0, 3) − .8600 are the weights. Such models are used by market participants both to design arbitrage strategies in proprietary trading desks, or to value portfolios of derivatives for trading or accounting purposes, or to determine hedge ratios for risk management reasons. The weights in column 6 equal the discount cash flows in column 5 divided by the price. Because they make a profit from the spread between them, the ask price is higher than the bid price. However, the trade is risky, because if interest rates move up, then the arbitrageur will suffer a decrease in value in the long position that is not paralleled by an equivalent decrease in value in the short (borrowing) position. The rates THE REPO MARKET 15 available are very similar to the Eurodollar rates (see Table 1.2). Figure 2.2 plots the time series of expected inflation from 1953 to 2008. In order to receive $100 overall, we must also be long two zero coupon bonds. 4 Basic Refinements in Interest Rate Risk Management. Finally, Chapter 8 discusses the residential mortgage backed securities market, in terms of the types of securities as well as their riskiness from an investment perspective. Given the small cost of failing to deliver, the number of fails spiked in the last quarter of 2008. Similarly, a homeowner who financed the purchase of his or her home using a fixed rate mortgage also bought an option to pay back the mortgage whenever he or she likes. Equation 2.13 be used to estimate the discount factors Z(t, T) for every maturity. This methodology is called the bootstrap methodology. The bankruptcy of Orange County is a classic example of the risk that is inherent in interest rate securities, and we will discuss this case study more thoroughly in Chapters 3 and 4 after we introduce some tools to measure interest rate risk. Exercises 7 to 12 use the two yield curves at two moments in time in Table 3.7, and the following portfolio: • Long $20 million of a 6-year inverse floaters with the following quarterly coupon: Coupon at t = 20% − r 4 (t − 0.25) where r4 (t) denotes the quarterly compounded, 3-month rate. In summary, an investor in a mortgage backed security obtains a legal claim to the cash flows (coupons) that are paid by the original homeowner. It is in the interest of banks to maintain their reserves as close to the limit as possible. These unexpected variations in cash flows make mortgage-backed securities risky and, for this reasons, such securities typically offer an additional return on investment, compared to Treasury securities. Define the weights wi = c/2 × Pz (0, Ti) for i = 1, .., n − 1 Pc (0, Tn) wn = (1 + c/2) × Pz (0, Tn) Pc (0, Tn) Then, the duration of the coupon bond is Dc = = n i=1 n wi Dz,Ti (3.12) wi Ti (3.13) i=1 That is, the duration of a coupon bond is a weighted average of coupon payment times T i . There is one final remark I want to make in regard to an introductory course in fixed income. However, these arbitrage opportunities are seldom exploitable: As soon as an arbitrageur tries to set up an arbitrage like the one described above, prices move instantly and the profit vanishes. In addition, we develop for this class of models the Monte Carlo simulation approach to pricing and hedging securities, a methodology widely used by market participants. Assume that maturities are at regular intervals of six months, that is, T 1 = t + 0.5 and Ti = Ti−1 + 0.5. Then, the bootstrap methodology to estimate discount factors Z(t, T i) for every i = 1, ..., n is as follows: 1. Indeed, note that we did not use only the ten bonds in Table 2.7 to fit the Nelson Siegel model, but the whole of 161 bonds with maturity less than five years in Table 2.6. Can the Nelson Siegel model fit all of these data reasonably well? Columns (6) and (7) report the present value and duration, respectively, of the 4%, 30-year T-bond that is used in the immunization strategy. What is the impact of a one basis point parallel shift of the term structure on the value of the portfolio? Continuous compounding refers to the limit in which payments accrue every instant. The text is filled with real-world examples and case studies, which I use to show step by step the fair valuation of most securities, the return an investor should expect from an investment, and the riskiness of such an investment. This coupon will depend on the 6-month rate at time t = 0.5, which we do not know today. 7.3.2 The Expectation Hypothesis 7.3.3 Predicting Excess Returns 7.3.4 Conclusion Coping with Inflation Risk: Treasury Inflation-Protected Securities 7.4.1 TIPS Mechanics 7.4.2 Real Bonds and the Real Term Structure of Interest Rates 7.4.3 Real Bonds and TIPS 7.4.4 Fitting the Real Yield Curve 7.4.5 The Relation between Nominal and Real Rates Summary 239 241 242 243 244 244 247 250 254 255 257 259 261 261 264 264 267 267 268 271 x CONTENTS 7.6 7.7 7.8 8 Exercises Case Study: Monetary Policy during the Subprime Crisis of 2007 2008 7.7.1 Problems on the Horizon 7.7.2 August 17, 2007: Fed Lowers the Discount Rate 7.7.3 September - December 2007: The Fed Decreases Rates and Starts TAF 7.7.4 January 2008: The Fed Cuts the Fed Funds Target and Discount Rates 7.7.5 March 2008: Bearn Stearns Collapses and the Fed Bolsters Liquidity Support to Primary Dealers 7.7.6 September – October 2008: Fannie Mae, Freddie Mac, Lehman Brothers, and AIG Collapse Appendix: Derivation of Expected Return Relation 272 275 276 280 280 281 281 282 282 BASICS OF RESIDENTIAL MORTGAGE BACKED SECURITIES 285 8.1 285 287 288 289 290 293 294 295 296 297 300 302 305 306 309 310 314 317 318 8.2 8.3 8.4 8.5 8.6 8.7 8.8 Securitization 8.1.1 The Main Players in the RMBS Market 8.1.2 Private Labels and the 2007 - 2009 Credit Crisis 8.1.3 Default Risk and Prepayment in Agency RMBSs Mortgages and the Prepayment Option 8.2.1 The Risk in the Prepayment Option 8.2.2 Mortgage Prepayment Mortgage Backed Securities 8.3.1 Measures of Prepayment Speed 8.3.2 Pass-Through Securities 8.3.3 The Effective Duration of Pass-Through Securities 8.3.4 The Negative Effective Convexity of Pass-Through Securities 8.3.5 The TBA Market Collateralized Mortgage Obligations 8.4.1 CMO Sequential Structure 8.4.2 CMO Planned Amortization Class (PAC) 8.4.3 Interest Only and Principal Only Strips. In well-functioning markets in which both the coupon bond Pc (t, Tn) and the zero coupon bonds P z (t, Ti) are traded in the market, if Equation 2.13 did not hold, an arbitrageur could make large risk-free profits. Treating equity as a portfolio, we obtain A L DE = (3.44) × DA − × DL A−L A−L $ $ = ADA = LDL = DL results in a duration mismatch problem, and Therefore, DA variation of interest rates affect the value of equity. The effective Federal funds rate is the size-weighted average rate of interest that banks charge to each other to lend or borrow reserves at the Federal Reserve. In particular, the 90-day Eurodollar rate has become a standard reference to gauge the conditions of the interbank market. If the dip of the 2.5 year yield was a data error, it gets corrected in the minimization of errors. On the same date, the 1 year to maturity, 2.75% Treasury note, was trading at $99.2343. That is, we have the formula Invoice price = Quoted price + Accrued interest (2.37) The quoted price is sometimes referred to as the clean price while the invoice price is sometimes also called dirty price. The idea is to resell its mortgages, now on the asset side of the bank, for cash. The discount factor is Z (t, T) = 0.97477. The two forces move in opposite directions (more cash flows imply higher prices, while the additional discount imply lower price). This investor can project himself to time t = 0.5, six months before maturity. What will this financial turmoil do to fixed income markets around the world? Additional Material for Students : Wiley Students' Companion Site. Finally, corporate debt has increased dramatically in the past few years, with a debt value of about $6.2 trillion. In addition, the Federal Reserve has been encouraging market participants to adopt a similar charge as part of best practices in repo market transactions. 3.2 DURATION The examples above calls for (a) a systematic methodology to assess the riskiness of a bond portfolio to movements in interest rates; and (b) a methodology to effectively manage such risk. Mortgage backed securities allow a bank to diversify this risk. Chapter 8 discusses this market in more detail. 61 CASE STUDY: ORANGE COUNTY INVERSE FLOATERS With the tools we have developed in this chapter we can price all sorts of securities. In this case, we obtained PIF (0, 3) = $116.28, Pz (0, 3) = 87.45, Pc (0, 3) = 128.83, and PF R (0, 3) = 100. To understand the intuition, we need to note that y correctly measures the expected return on an investment only under the strict condition that the investor can reinvest all of the coupons at the rate y over the life of the bond. 5 That √ 2 is, f (x) = 1/ 2π × e−x /2 and N (x) = x −∞ f (y)dy. Chapter 8 describes this market in detail as well as the type of securities that are exchanged 4 See the "Guide to FR2004 Settlement Fails Data," Federal Reserve of New York. To convince ourselves that this dynamic immunization strategy works, we can repeat the above exercise many times, for many interest rate scenarios, and plot the histogram of the final value W T . The duration of this portfolio is then 5, implying that a one basis point increase in interest rate decreases 78 BASICS OF INTEREST RATE RISK MANAGEMENT Figure 3.2 First Derivative of a Zero Coupon Bond with Respect to Interest Rate r 100 90 80 Zero Coupon Bond Pz(r,t;T) 70 60 50 40 dr 30 dP 20 Slope = d P / d r 10 0 0 5 10 15 Interest Rate r the value of the portfolio approximately by dP ≈ −DP × P × dr = −5 × $100 million × .01% = −$50, 000 3.2.2 Duration of a Portfolio What is the duration of a portfolio of securities? 107 3.7.6 Conclusion 108 Case Analysis: The Ex-Ante Risk in Orange County's Portfolio 108 3.8.1 The Importance of the Sampling Period 109 3.8.2 Conclusion 110 Appendix: Expected Shortfall under the Normal Distribution 111 BASIC REFINEMENTS IN INTEREST RATE RISK MANAGEMENT 113 4.1 113 116 118 118 120 121 Convexity 4.1.1 The Convexity of Zero Coupon Bonds 4.1.2 The Convexity of a Portfolio of Securities 4.1.3 The Convexity of a Coupon Bond 4.1.4 Positive Convexity: Good News for Average Returns 4.1.5 A Common Pitfall viii CONTENTS 4.2 4.3 4.4 4.5 4.6 5 4.1.6 Convexity and Risk Management 4.1.7 Convexity Trading and the Passage of Time Slope and Curvature 4.2.1 Implications for Risk Management 4.2.2 Factor Models and Factor Neutrality 4.2.3 Factor Duration 4.2.4 Factor Neutrality 4.2.5 Estimation of the Factor Model Summary Exercises Case Study: Factor Structure in Orange County's Portfolio 4.5.1 Factor Estimation 4.5.2 Factor Duration of the Orange County Portfolio 4.5.3 The Value-at-Risk of the Orange County Portfolio with Multiple Factors Appendix: Principal Component Analysis 4.6.1 Benefits from PCA 4.6.2 The Implementation of PCA 122 126 127 129 130 132 134 136 137 138 142 142 142 144 145 149 150 INTEREST RATE DERIVATIVES: FORWARDS AND SWAPS 153 5.1 154 157 158 161 162 164 167 169 170 171 171 174 175 176 178 179 181 182 184 184 189 191 5.2 5.3 5.4 5.5 5.6 5.7 5.8 Forward Rates and Forward Discount Factors 5.1.1 Forward Rates by No Arbitrage 5.1.2 The Forward Curve 5.1.3 Extracting the Spot Rate Curve from Forward Rates Forward Rate Agreements 5.2.1 The Value of a Forward Rate Agreement Forward Contracts 5.3.1 A No Arbitrage Argument 5.3.2 Forward Contracts on Treasury Bonds 5.3.3 The Value of a Forward Contract Interest Rate Swaps 5.4.1 The Value of a Swap 5.4.2 The Swap Rate 5.4.3 The Swap Curve 5.4.4 The LIBOR Yield Curve and the Swap Spread 5.4.5 The Forward Swap Contract and the Forward Swap Rate 5.4.6 Payment Frequency and Day Count Conventions Interest Rate Risk Management using Derivative Securities Summary Exercises Case Study: PiVe Capital Swap Spread Trades 5.8.1 Setting Up the Trade CONTENTS 5.8.2 5.8.3 5.8.4 6 191 193 196 INTEREST RATE DERIVATIVES: FUTURES AND OPTIONS 199 6.1 199 200 202 203 205 208 209 213 220 223 223 225 226 233 6.2 6.3 6.4 6.5 7 The Quarterly Cash Flow Unwinding the Position? We can substitute everything into Equation 3.50, to find DInverse = wZero × Dzero + wF ixed × DF ixed + wF loating × DF loating = 0.7521 × 3 + 1.1079 × 2.6685 − .8600 × 1 = 4.35 The duration of the 3-year inverse floater is 4.35. The first discount factor Z(t, T 1) is given by Z(t, T1) = Pc (t, T1) 100 × (1 + c1 /2) (2.26) 7 Notice a little approximation in this computation: The T-note would pay its coupon on December 31, 2005, rather than December 29. 76 BASICS OF INTEREST RATE RISK MANAGEMENT The duration of the asset is then defined as 3 Duration = D P = − 1 dP P dr (3.1) The shift dr is a small uniform change across maturities, such as, for instance, 1 basis point: dr = .01%. As we discuss in this and Chapter 3, it is exactly in 1994 that Orange County, a rich county in California, lost $1.6 billion and went bankrupt. APPENDIX: EXTRACTING THE DISCOUNT FACTORS Z(0, T) FROM COUPON BONDS 65 floaters discussed earlier is that the parity of floating rate to fixed rate is greater than one. You are given the following data on different rates with the same maturity (1.5 years), but quoted on a different basis and different compounding frequencies: • Continuously compounded rate: 2.00% annualized rate • Continuously compounded return on maturity: 3.00% • Annually compounded rate: 2.10% annualized rate • Semi-annually compounded rate: 2.01% annualized rate You want to find an arbitrage opportunity among these rates. For instance, they were all relatively low in the 1960s, 1 The spot rate curves are calculated by fitting the extended Nelson Siegel model to coupon bond data from CRSP (Monthly Treasuries) ©2009 Center for Research in Security Prices (CRSP), The University of Chicago Booth School of Business. Market participants refer to the most recently issued Treasury securities as on-the-run securities, while all the others are called off-the-run. The intuition is that any additional periods increase the cash flow by c/2 while they also increases the discount rate by the same amount r2 /2. We will talk about TIPS more exhaustively in Chapter 7. Then, the first derivative of F with respect to x, denoted by dF/dx, is given by Derivative of F (x) with respect to x = dF = A × a × eax = a × F (x) dx (3.3) An example of the function F (x) is the zero coupon bond formula studied in Chapter 2 Pz (t, T) = 100 × Z(t, T) = 100 × e−r(T −t) . Specifically, the discount function is given by Z (t, T) = 1 + L a f (T) (2.57) =1 What are the functions f (T)? 30% in T−Bond D. LIBOR is however one of the most important benchmark rates, used often as the reference index in the large over-the-counter derivatives market. 5 We are using the continuously compounded interest rate r(t, T) to describe the curve. If the interest rate increases, the future cash flow increases, but it is discounted by a higher rate. Not only does this choice allow for simpler formulas, as we showed in the previous sections, but it also implies that the durations of different assets are defined against the variation of the same interest rates, namely, the spot rates. For instance, European investors who purchased safe U.S. Treasury notes in 2005 have been hurt by the devaluation of the dollar with respect to the euro between 2006 and 2008. No claim of wrongdoing by any party is made here. Considering that the mortgage backed securities market has become the dominant fixed income market in the U.S. (its value as of December 2008 is about $9 trillion, compared to only $6 trillion of the U.S. debt), the understanding of the impact of options on fixed income instruments has never being more important. That is, there is only 5% probability that the portfolio losses will be higher than $3 million. Curve fitting treats this latter problem. 2. VaR is a statistical measure of risk, and as with any other statistical measure, it depends on distributional assumptions and the sample used for the calculation. Moreover, there is not a dominant market: What we called a derivative market in the past is now larger in sheer size than the primary market. For now, we only consider the effect of leverage. The weighted average of cash flow maturities in Equation 3.19 is called the Macaulay duration DM c = n wj × Tj (3.20) j=1 We will rarely use the variation in the semi-annually compounded yield to maturity for risk management purposes, and rather use the variation in the continuously compounded spot curve. What about agency mortgage backed securities, which are guaranteed by the U.S. government as well? 59 CASE STUDY: ORANGE COUNTY INVERSE FLOATERS Table 2.4 Yield Curve on March 15, 2000 Maturity Yield Maturity Yield Maturity Yield 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00 2.25 2.50 6.33% 6.49% 6.62% 6.71% 6.79% 6.84% 6.87% 6.88% 6.89% 6.88% 2.75 3.00 3.25 3.50 3.75 4.00 4.25 4.50 4.75 5.00 6.86% 6.83% 6.80% 6.76% 6.72% 6.67% 6.62% 6.57% 6.51% 6.45% 5.25 5.50 5.75 6.00 6.25 6.50 6.75 7.00 7.25 7.50 6.39% 6.31% 6.24% 6.15% 6.05% 5.94% 5.81% 5.67% 5.50% 5.31% Yields calculated based on data from CRSP (Daily Treasuries). The advice and strategies contained herin may not be suitable for your situation. The following example illustrates the point. Financial institutions, corporations, and even governments use swaps (a) to change the sensitivity of their cash flows to fluctuations in interest rates; (b) to alter the timing of their payments and revenues; (c) or even simply for investment purposes within complex trading strategies. The duration of a floating rate bond with annual coupons is equal to the time of the first coupon at reset dates. Panel C of Figure 3.3 plots the histogram of the changes in the portfolio i.e., the portfolio profits and losses (P&L). The financial crisis of 2007 - 2009 led the Federal Reserve to lower the reference Fed funds rate to close to zero, and repo rates also fell to essentially zero. In this case, the coupon rate at time T = 1 is c(1) = 100 × r2 (0.5)/2 = 103, and the value of the bond at t = 0.5 is Value bond at 0.5 = Present value of (100 + c(1)) = 100 + 3 = 100 1 + 0.06/2 Still the same round number, equal to par. The idea here, however, is to provide a methodology, and not a shopping list. ... We will regard such situations as "noise", that is, a little imprecision in market prices due to liquidity or external factors that sometimes impede the smooth functioning of capital markets. To see this, let us consider the dotted line at the bottom of the plot, and the dashed line at the top. Their feedback on earlier versions of the manuscript was invaluable. These securities became very popular during the Orange County bankruptcy at the end of 1994, since it is estimated that a significant fraction of the county's portfolio comprised these securities. The trader is now lending money to the repo dealer against the bond. 104 3.7.2 The Risk in Leverage 105 3.7.3 The Risk in Inverse Floaters 105 3.7.4 The Risk in Leveraged Inverse Floaters 106 3.7.5 What Can We Infer about the Orange County Portfolio? In order to improve the liquidity and to mitigate credit risk, the market evolved into one in which many similar mortgages are pooled together to form a large collateral of assets. This representation is derived from the basic one above, but it can be useful nonetheless to report it: Pc (t, Tn) = n c/2 × 100 2×(Ti −t) i=1 (1 + r2 (t, Ti) /2) + 100 2×(Tn −t) (1 + r2 (t, Tn) /2) (2.15) A useful fact is the following: Fact 2.9 Let the semi-annual discount rate be constant across maturities, r 2 (t, Ti) = r2 for every Ti . What is the difference between these two strategies? To reduce or eliminate this maturity mismatch, the financial firm may alter the composition of its portfolio. Now use the February 15, 1994 yield curve to price the stream of cash flows on May 15, 1994. However, it does not say anything about how large the losses could be if they do occur. Chapter 7 links the fixed income market to the real economy. T-notes have maturity up to ten years, while bonds up to thirty years. Different from zero coupon bonds, the duration of the coupon bond is shorter than its maturity. But often the gap across maturities may span longer periods, in which case the bootstrap methodology does not work well. In Chapter 7 we look at various explanations that economists put forward to account for the behavior of discount factors and interest rates. These macro economic conditions affect the relative supply and demand of Treasury securities and thus their prices. Let the market rates available to the two firms be the ones in Table 1.7. That is, Firm A can either borrow at a fixed coupon rate of 15% or at a floating rate with coupon linked to the six-month LIBOR rate plus 3%. For one, money today can be put in a safe place (a bank or under the mattress) until tomorrow, while the opposite is not easily doable. If Ms. Caselli is going to consume only the interest on her investment, strategy 1 is more risky than strategy 2. Let x = 2.8/20.5 = 0.1366 be the fraction of total assets invested in leveraged inverse floaters. Mortgage backed securities market: This is the largest debt market in United States. Many options are implicit in many securities. From the very beginning we will see that it is actually hard to apply the simple formulas of fixed income models, even the most elementary ones such as a present value formula, to real-world data. Indeed, the immunization strategy effectively ensures that the losses on the cash investment due to a declining interest rates are compensated by the capital gains on the long-term bond. From Section 2.5, we recall that the value of a floating rate bond is always equal to par at reset dates. Consider two STRIPS: One has 3-years to maturity and yields a continuously compounded rate r(0, 3) = 10%, while the second has 5 years to maturity and yields a continuously compounded rate r(0, 5) = 5%. 3.4 ASSET-LIABILITY MANAGEMENT Asset liability management is the most classic example of interest rate risk management. The point of this calculation, however, is that the portfolio of Orange County may well have been invested only in short-term zero coupon bonds with maturity 2.89. The duration of the floating rate bond is 6 months, as discussed in Example 3.5. Thus, using Equation 3.22, we find Dollar duration of P c(0, T) = = Dollar duration of P F R (0, T) = = $100 × Duration of fixed rate bond $100 × 8.34 = $834 $100 × Duration of floating rate bond $100 × 0.5 = $50 Thus, the dollar duration of the long-short portfolio is Dollar duration of long-short portfolio = $834 − $50 = $784 (3.24) Again using Equation 3.21, we have that one basis point increase in interest rate dr generates change in the long-short portfolio: $ × d r = −$784 × .01/100 = −.0784 (3.25) Change in portfolio value = d W = −D W That is, the long-short portfolio with trade size of $1 million, for instance, stands to lose $78,400 for every basis point increase in the level of interest rates. An alternative is to look at all bonds that are being quoted in the market today

(December 31, 1993) and use this data to plot the yield curve. I show the additional flexibility offered by these multifactor models to price interesting additional structured notes and derivative securities, such as those that depend on multiple points of the term structure. The rules of no arbitrage determine the relative pricing across fixed income securities and explain their high correlation. The liability is given by the repo transactions, which are financed at the overnight rate. Interest rates: The promised rate of return of an investment, an interest rate needs a compounding frequency to be well defined. To understand the logic, consider next example, which entails a reverse repo, whose rates are also quoted in Table 1.2. EXAMPLE 1.2 Consider a trader who thinks a particular bond, such as the on-the-run 30-year Treasury bond, is overpriced and wants to take a bet that its price will decline in the future. The availability of these zero coupon bond securities with maturity up to 30 years enables investors to be more effective in their investment strategies and in their risk management practices, as we will discuss in later chapters. EXAMPLE 2.10 On the same date, t = June 30, 2005, the December 31, 2006 Treasury note, with coupon of 3%, was trading at $99.1093. The notion is that these quantities are all highly correlated with the same events. Copyright 29 30 BASICS OF FIXED INCOME SECURITIES EXAMPLE 2.1 On August 10, 2006 the Treasury issued 182-day Treasury bills. Given a discount curve Z(0, T), we can compute its duration by following the calculations in Table 3.1. In this table, the second and third columns present the payment times and the payment amounts. For instance, a commercial bank collects deposits – a short-term liability whose interest rate changes daily – to make medium- and long-term loans to other business or households. In short, the discount factor Z(t, T) records the time value of money between t and T . 1 The various lines, all very close to each other, are the continuously compounded yields of zero coupon bonds for maturities from 1 month to 10 years. EXAMPLE 2.2 On August 10, 2006 the U.S. government also issued 91-day bills with a maturity date of November 9, 2006. 3.8.1 The Importance of the Sampling Period The VaR calculation is very sensitive to the sample used in the calculation. The chapter also illustrates the use of this model for the pricing of options. What happens then at time T ? As discussed in Chapter 1, the government issues a number of securities, such as Treasury bills, notes and bonds, to investors, receiving money today in exchange for money in the future. The rationale behind its default safety is that these bonds are backed by the ability of the U.S. government to levy taxes on its citizens in the future to pay the debt back. To take this case into account we need to develop further tools, as we will do in Parts II and III of this book. Cash flows that arrive sooner rather than later are less sensitive to changes in interest rates (for instance, a cash flow arriving tomorrow has no sensitivity to interest rates). In other words, it implies that investors prefer to have $100 dollar in six months rather than in three months, violating the principle that agents prefer to have a sum of money earlier rather than later. Column (4) reports the present value of the annuity, assuming that the term structure is flat and equal to the interest rate in Column (2). DURATION 89 The problem turns out to be especially severe for portfolios that include derivative securities, either implicitly or explicitly. Two compounding frequencies are particularly important: semi-annual compounding and continuous compounding. COUPON BONDS 51 2.4.4.1 Treasury Bills. This is done in Figure 3.4. As it can be seen, the strategy works well, as the final wealth is always positive. Hence, the return on capital is PT − Pt − Repo interest Return on capital for trader = Haircut The position is highly leveraged and entails quite large risks. 1.5 THE MORTGAGE BACKED SECURITIES MARKET AND ASSET-BACKED SECURITIES MARKET One of the interesting patterns evident in Figure 1.1 is the dramatic growth experienced by the mortgage backed securities market, which hit the $8.9 trillion mark by the end of 2008. STRIPS (Separate Trading of Registered Interest and Principal Securities) are zero coupon bonds created from available U.S. Treasury notes and bonds by splitting the principal and each of the coupons from the bond. CASE ANALYSIS: THE EX-ANTE RISK IN ORANGE COUNTY'S PORTFOLIO 109 From the data in the top panel of Figure 3.7 we can compute the historical mean and standard deviation of dr, and thus obtain the mean and standard deviation of dP . 10… Rate Derivatives and VolatilityA. In addition, we should add the large swap market, now the main reference market for fixed income security dealers, which had a global market value of about $8 trillion in 2008. (updated versions also available here) Some Chapters Preview : Google Preview of Fixed Income Securities by Pietro Veronesi. What if it is 5% or 7%? In particular, Fact 2.12 shows that if T i denotes the last reset date, Ti+1 denotes the next reset date, t is the current time, with T i < t < Ti+1 , then the price of a floating rate bond with maturity T and semi-annual payments is given by (see Equation 2.40) PF R (t, T) = Z(t, Ti+1) × 100 × [1 + r2 (Ti)/2] (3.14) where r2 (Ti) is the reference rate that is determined at the last reset date. I now describe the three parts of the book in more detail. EXAMPLE 2.8 In Example 2.7, suppose that the 2-year note was trading at $98. However, a number of other problems arise with these functions, the most important being the decision of how many knot points to include and, in addition, where to position them. 3 Basics of Interest Rate Risk Management. Although they appear the same, it is crucial to note that the yield to maturity y is defined as the particular constant rate that makes the right-hand side of Equation 2.30 equal to the price of the bond. Eurodollar deposit rates at various maturities; 4. The same reasoning applies more generally. Assume that the zero coupon yield curve is flat at 4% (semi-annually compounded), so that a 30-year T-bond with 4% coupon trades at par. 1.2.1 Zero Coupon Bonds Zero coupon bonds are securities that pay only the principal at maturity. Here, we must remember that an inverse floater is given by a portfolio of more basic securities, of which we can compute the duration easily. Second, the time 0 of the valuation is a reset date. In this book we will see how these rules of no arbitrage allow us to both compute the fair value of fixed income instruments and to investigate their relative prices. The use of examples and case studies, as well as end-of-chapter exercises enables readers to grasp these differences and understand why one or another model may be useful in one or another circumstance. The VaR measure is based on the volatility of the underlying assets in the portfolio. In addition, options are implicitly embedded in several other securities, such as callable bonds, mortgage backed securities, and other types of structured notes. Banks and financial institutions have various means of borrowing and lending at any point in time. The chapter also discusses the basic bond pricing formula, as well as some important methodologies for extracting discounts from observable bond prices. The reason is that a trader that has to deliver a given Treasury security to a conterparty is willing to pay to get hold of the security rather than incurring the penalty. Sometimes I use examples to motivate new important concepts. The British Bankers Association publishes daily the LIBOR rates. Chapter 4 contains some refinements in the risk management techniques introduced in Chapter 3: In particular, the chapter illustrates the notion of bond convexity, and its implication for risk and risk management, as well as the concepts of yield curve's slope and curvature dynamics. For instance, a corporation that issues a floating rate bond – a bond whose coupon is tied to the level of a short-term floating rate – may be worried about an interest rate hike in the future, a scenario that may drain too much financial resources from the coporation. That is, we can mentally think of continuous compounding as the daily compounding frequency. Before we can answer this important question, however, we need to recall the following two concepts from calculus. 3.7.4 The Risk in Leveraged Inverse Floaters Recall that a leveraged inverse floater has a coupon that moves (inversely) to interest rates by more than one-to-one. The rate of return is indeed the difference between payoff and initial investment, divided by the latter. In this case, the following is true Fact 2.11 If the spread s = 0, the ex-coupon price of a floating rate bond on any coupon date is equal to the bond par value. This gives Present value of total payoff at T n c × 100 2×(T −Ti) (1 + y/2) + 100 = Z (t, T) × × 2 i=1 n (1 + y/2)2×(T −Ti) 100 c × 100 + = × 2×(T −t) 2 (1 + y/2) (1 + y/2)2×(T −t) i=1 n 1 c × 100 100 = + × 2×(Ti −t) 2×(T −t) 2 (1 + y/2) (1 + y/2) i=1 y = P (t, T) We find then that the price of the bond is (by definition) equal to the present value of the total payoff at T , discounted at the yield to maturity y, under the assumption that all of the coupons can be reinvested at same rate y over the life the of the bond. As we saw in this chapter, in absence of arbitrage opportunities, any bond with coupon c that matures in three years (even if it was issued, for example, seven years ago), must have the same price as a 3-year bond, issued today, with coupon c. This site contains a set of Excel spreadsheets related to examples in the book. Table 2.3 contains the description of computations for the valuation of a 2-year floating rate bond. The lessons learned in this chapter will help us understand what type of exposure Orange County had in its portfolio that could lead to a loss of this magnitude. Cubic splines are the most used functions (so, third order), as they generate smooth forward curves. Are the prices the same as what the market says? This is the continously compounded interest rate. Recall Equation 3.2: (3.28) dP = −DP × P × dr Given the value of the portfolio P and its duration D P , we can transform the probability distribution of interest rate changes dr into the probability distribution of portfolio changes dP , and from the latter, we can compute the portfolio potential losses. 2.4.2 From Coupon Bonds to Zero Coupon Bonds We can also go the other way around: If we have enough coupon bonds, we can compute the implicit value of zero coupon bonds from the prices of coupon bonds. In particular, the U.S. government issues Treasury notes, which are fixed income securities with maturity up to 10 years; Treasury bonds, which have maturities up to 30 years; and TIPS (Treasury Inflation Protected Securities), which have coupons that are not constant, but rather are linked to a recent inflation rate figure. The chapter also introduces the popular risk measures of Value-at-Risk and expected shortfall. Chg Asked Yield 0.001 0.6 -0.068 1.31 -0.056 1.59 -0.146 1.72 -0.194 1.78 -0.236 1.83 -0.277 1.93 -0.318 2 -0.359 2.05 -0.413 2.23 -0.442 2.23 -0.539 2.42 -0.481 2.34 -0.48 2.78 -0.515 2.69 -0.566 2.94 -0.589 2.97 -0.607 3.07 -0.647 3.1 -0.666 3.11 -0.725 3.18 -0.763 3.26 -0.814 3.29 -0.828 3.36 -0.903 3.41 -0.462 3.58 -0.489 3.6 -0.502 3.64 -0.525 3.73 -0.538 3.77 -0.573 3.9 -0.599 3.92 -0.707 4.06 -0.483 4.09 -0.491 4.14 -0.482 4.21 -0.505 4.23 -0.468 4.21 -0.474 4.23 -0.559 4.3 -0.565 4.32 -0.601 4.4 -0.609 4.4 -0.596 4.46 -0.602 4.47 -0.312 4.51 -0.314 4.54 -0.316 4.56 -0.319 4.58 -0.373 4.61 -0.393 4.62 -0.396 4.64 -0.416 4.65 -0.454 4.66 -0.475 4.67 -0.477 4.68 -0.479 4.7 -0.481 4.71 -0.484 4.7 Maturity Year Month Day 2023 8 15 2023 11 15 2024 2 15 2024 5 15 2024 8 15 2024 11 15 2025 2 15 2025 5 15 2025 8 15 2025 11 15 2026 2 15 2026 5 15 2026 8 15 2026 11 15 2027 2 15 2027 5 15 2027 8 15 2027 11 15 2028 2 15 2028 5 15 2028 8 15 2028 11 15 2029 2 15 2029 5 15 2029 8 15 2029 11 15 2030 2 15 2030 5 15 2030 8 15 2030 11 15 2031 2 15 2031 5 15 2031 8 15 2031 11 15 2032 2 15 2032 5 15 2032 8 15 2032 11 15 2033 2 15 2033 5 15 2033 8 15 2033 11 15 2034 2 15 2034 5 15 2034 8 15 2034 11 15 2035 2 15 2035 5 15 2035 8 15 2035 11 15 2036 2 15 2036 5 15 2036 8 15 2036 11 15 2037 2 15 2037 5 15 2037 8 15 2038 2 15 Bid 50.039 49.424 48.815 48.286 47.746 47.194 46.797 46.221 45.537 44.972 44.357 43.879 43.332 42.828 42.445 41.934 41.467 41.025 40.685 40.216 39.694 39.178 38.686 38.277 37.815 37.493 37.138 36.749 36.463 36.084 35.691 35.282 34.957 34.656 34.438 34.264 33.918 33.556 33.337 32.984 32.674 32.329 32.008 31.651 31.357 31.008 30.662 30.321 29.983 29.649 29.339 29.268 28.865 28.664 28.249 28.212 27.901 27.331 Ask 50.059 49.444 48.835 48.306 47.766 47.214 46.817 46.241 45.557 44.992 44.377 43.899 43.352 42.848 42.465 41.954 41.487 41.045 40.705 40.236 39.714 39.198 38.706 38.297 37.835 37.513 37.158 36.769 36.483 36.104 35.711 35.302 34.977 34.676 34.458 34.284 33.938 33.576 33.357 33.004 32.694 32.349 32.028 31.671 31.377 31.028 30.682 30.341 30.003 29.669 29.359 29.288 28.885 28.684 28.269 28.232 27.921 27.351 Chg Asked Yield -0.487 4.7 -0.489 4.71 -0.529 4.71 -0.532 4.71 -0.553 4.7 -0.555 4.7 -0.408 4.69 -0.371 4.69 -0.372 4.71 -0.297 4.72 -0.297 4.73 -0.298 4.72 -0.298 4.73 -0.299 4.73 -0.262 4.71 -0.263 4.72 -0.263 4.71 -0.206 4.71 -0.11 4.69 -0.11 4.69 -0.111 4.7 -0.111 4.71 -0.092 4.71 -0.033 4.71 -0.033 4.71 -0.015 4.69 -0.015 4.68 -0.015 4.68 0.063 4.66 0.063 4.66 0.063 4.65 -0.074 4.65 -0.074 4.64 -0.074 4.63 -0.074 4.61 -0.075 4.58 -0.075 4.58 -0.075 4.57 -0.076 4.55 -0.075 4.55 -0.076 4.54 -0.076 4.54 -0.075 4.54 -0.075 4.54 -0.076 4.53 -0.075 4.53 -0.076 4.53 -0.075 4.53 -0.075 4.53 -0.075 4.53 -0.075 4.53 0.161 4.49 0.389 4.5 0.161 4.49 0.384 4.5 0.161 4.47 0.386 4.47 0.23 4.46 14 AN INTRODUCTION TO FIXED INCOME MARKETS month T-bills (BOT). For instance, a 10-year STRIP provides the certain annualized return in Equation 2.29 if the security is held until maturity. More complex models take into account more features of the data, but they are also harder to implement. This is arbitrary. Given that it is practically impossible for an investor to reinvest all of the coupons at the constant yield to maturity y , this latter measure is in fact a poor measure of expected return. In addition, exercises often require students to carry out a risk analysis, by computing hedge ratios or risk measures. cn (T1) cn (T2) ... Given Equation 2.13, we can write the value of the two 46 BASICS OF FIXED INCOME SECURITIES securities as:7 Pbill (t, T1) = $98.3607 = $100 × Z(t, T1) (2.23) Pnote(t, T2) = $99.2343 = $1.375 × Z(t, T1) + $101.375 × Z(t, T2) (2.24) We have two equations in two unknowns [the discount factors Z(t, T 1) and Z(t, T2)]. In its pure form, an arbitrage opportunity is defined as follows: Definition 1.1 An arbitrage opportunity is a feasible trading strategy involving two or more securities with either of the following characteristics: 1. The use of a longer sample that takes into account such facts is therefore more conservative for the risk manager. That is, if we consider a different path of interest rates, would we still get a positive number? Using the data plotted in Panel A of Figure 3.3, we find that the monthly change in interest rate has mean µ = 6.5197 × 10 − 006 and stadard deviation σ = .4153%. Figure 2.7 plots both the bond prices (stars) and the model prices (diamonds) for the various maturities: The model works well if the stars are close to the diamonds. Fact 2.3 For a given final payoff, more frequent accrual of interest implies a lower interest rate figure. Still, compared to Treasury debt securities mortgage-backed securities have many peculiarities regarding the timing of promised cash flows, which may vary unexpectedly due changes in interest rates, or changes in housing prices, or a severe recession. The methodology is illustrated through various real-world examples, as well as a case study at the end of the chapter which features real data, and demonstrates the methodology in action. In addition to the potential capital losses in the bond price during a shorter period of time than the bond's maturity, an investment in U.S. Treasury securities entails additional sources of risk. Each model has pros and cons and there is always a tradeoff between using one or another model. 24 AN INTRODUCTION TO FIXED INCOME MARKETS Figure 1.8 A Swap Deal Firm A Swap: 11% ========== Swap: LIBOR Firm B ⇓ ⇓ ⇓ ⇓ ⇓ ⇓ Bond Market Bond Market Bond A: coupon = LIBOR + 3% Bond B: coupon = 12% Consider now the net cash flow from each of these firms when we take together the bond issuance and the swap deal. 1 To do so, governments will need to borrow even more than in the past, thereby increasing government debt and thus affecting 1 There is much disagreement on whether such fiscal stimulus will in fact work. Figure 1.4 provides a schematic representation of the repo transaction: At time t, the trader buys the bond at market price Pt and enters a repurchase agreement with the repo dealer. The term structure of interest rates is defined as follows: Definition 2.3 The term structure of interest rates, or spot rate curve, or yield curve, at a certain time t defines the relation between the level of interest rates and their time to maturity T − t. Finally, this book exists also because my editor, Judith Joseph of Wiley, pushed me to write it. The following example illustrates the trade: EXAMPLE 1.1 Suppose that a trader on September 18, 2007 (time t) wants to take a long position until a later time T on a given U.S. security, such as the 30-year Treasury bond. That is, given two dates T1 and T2 with T1 < T2 , it is always the case that Z(t, T1) ≥ Z(t, T2) (2.1) The opposite relation Z(t, T 1) < Z(t, T2) would in fact imply a somewhat curious behavior on the part of investors. c1 (Tn) ⏐ c2 (T1) c2 (T2) ... For instance, z = 1.645 for the 95% VaR, and recall from Equation 3.30, dP ~ N (µP , σ P). Consider the reverse repurchase agreement displayed in Figure 1.5. At maturity of the repo contract (time T), the trader must return the bond to the repo dealer in exchange for the n cash amount Pt × (1 + 360 × repo rate). Vice versa, given a discount factor, we can define an interest rate together with its compounding frequency. However, there is little doubt that it will increase government debt. The capital losses on the investment would be approximately Capital losses ≈ 17.72 × $1 million × .03 = $531, 000 That is, a 3 percent increase in the interest rate may more than halve the savings of Ms. Caselli. Table 2.1 reports the n−times compounded interest rate also for more frequent compounding. Due to a series of unfortunate events, the investor in Exercise 2 just found out that he must raise $50 million. Many alternatives have been proposed. The chapter illustrates these concepts with a discussion of the (likely) risks embedded in the portfolio of Orange County, which lost $1.6 billion and declared bankruptcy in 1994. The number of securities described in this table is daunting. A reverse repo is the opposite transaction, namely, it is the purchase of the security for cash with the agreement to sell it back to the original owner at a predetermined price, determined, once again, by the repo rate. It is worth mentioning that although the expectation of future higher interest rates may determine today's term structure of interest rates, this is not the only channel. 108 BASICS OF INTEREST RATE RISK MANAGEMENT 3.7.6 Conclusion In conclusion, this case illustrates the risk embedded in fixed income securities, and, in particular, in leveraged positions. 1.3 THE MONEY MARKET When we speak of the money market, we refer to the market for short-term borrowing and lending. In this sense, the interpretation of duration as the weighted average of cash flow payments is strongly misleading. Apache/2.4.41 (Ubuntu) Server at studyabroad.sdbor.edu Port 443 FIXED INCOME SECURITIES Valuation, Risk and Risk Management Pietro Veronesi University of Chicago JOHN WILEY & SONS, INC. This example shows that even if the average time of future cash flows can be relatively long – a floating coupon bond with 10 years to maturity, for instance, has an average time of future payments of several years – the duration could be very small. 2.7 EXERCISES 1. They corresponds to the 3-month and 5-year interest rates, respectively. Assuming a flat term structure at a semi-annually compounded interest rate of 4%, the present value of this stream of cash flows is about $1,000,000: $1, 000, 000 = $28, 767 × 60 i=1 1 1+ 4% 2 i (3.37) 92 BASICS OF INTEREST RATE RISK MANAGEMENT where 60 is the number of payments. EXAMPLE 2.4 On March 1, 2001 (time t) the Treasury issued a 52-week Treasury bill, with maturity date T = February 28, 2002. The appropriate discount factors should then be used to discount future cash flows. This discussion emphasizes also the crucial difference between a rate of return on an investment, and an interest rate, which are related, but different, concepts. Mortgage backed securities are collateralized by pools of residential and non residential mortgages and sold to investors who then receive claims to the mortgages coupons. The big question is what is keeping the prices of the interest rate instruments tied to each other. Thus, the ratio between purchase price and the payoff, 0.97477 = $97.477/$100, can be considered the market-wide discount factor between the two dates August 10, 2006 and February 8, 2007. The compounding frequency is defined as follows: Definition 2.2 The compounding frequency of interest accruals refers to the annual number of times in which interest is paid and reinvested on the invested capital. From an investment perspective, a large part of the mortgage-backed securities market is considered default free, because these three big players – Ginnie, Freddie and Fannie – have an implicit or explicit backing of the full faith of the U.S. government. That is, rather than quoting a price P bill (t, T) for a Treasury bill, Treasury dealers quote the following quantity 100 − Pbill (t, T) 360 × (2.35) d= 100 n where n is the number of calendar days between t and T . Besides describing their properties and their pricing methodology, several examples throughout the chapter also illustrate the use of such derivative contracts for an effective risk management strategy. We can answer this question by using the concepts of Value-at-Risk and expected shortfall introduced in Sections 3.2.8 and 3.2.9, respectively. 16 Recall that the bid and ask prices are the quotes at which security dealers are ready to buy or sell the securities. Similarly, the next block of markets in Table 1.1 shows the interest rate derivatives markets. It is easiest to introduce the concept by looking at a concrete example. The semi-annually compounded interest rate r2 (t, T) defines a payoff at maturity T given by 2×(T −t) r2 (t, T) Payoff at T = Investment at t × 1 + . What is the intuition behind immunization strategies? To this end, I give my students challenging, data-oriented homeworks to make them aware not only of the vast possibilities offered by fixed income term structure models, and their usefulness to price, hedge or implement a risk analysis of a given security, but also to have students realize the limitations of such fixed income models, and the fact that models need data for their effective application to the real world. Luckily, the main concepts developed earlier readily extend to multifactor models. In particular, was there anything that ex ante could have warned Orange County's Treasurer and its creditors regarding the potential risk that the portfolio was bearing? In particular, they could consider the following swap deal: • Firm A pays B a fixed rate payment at 11% per year; and • Firm B pays A a floating rate payment at LIBOR. An interest rate model allows us to compute the price of one security by using a portfolio of other securities, so long as the latter is properly rebalanced over time as interest rates change. The difference in spreads between the two firms across asset classes, floating and fixed coupon bonds, generates the possibility of a trade. Descriptive material is from the case study ERISK: Orange County, downloaded from . Discount factors decrease with the time horizon and also vary over time. 14 An interesting security to price is an inverse floater. In particular, we find dr = 0.00432; Mean(dr) = µ dr = 4.71e − 05; Std(dr) = σ which implies dr = −0.0028; Std(dP) = DP × P × σ dr = 0.2563 Mean(dP) = −DP × P × µ Figure 3.8 also reports the normal density with mean µ P = −.0028 and standard deviation σ dP = 0.2563. It is the responsability of the investor who sold the bond short to make the regular coupon payments to the counterparty. 7 We 8 See are still making the simplifying assumption that we know that c(t) > 0 for sure, i.e. that r 1 (t) < 25%/2. This chapter concludes the first part of the book, which aims at providing some basic notions of fixed income securities. Any other discount factor Z(t, T i) for i = 2, ..., n is given by i−1 Pc(t, Ti) − ci /2 × 100 × j=1 Z(t, Tj)) Z(t, Ti) = 100 × (1 + ci /2) 47 (2.27) This procedure is relatively simple to implement, as the example above shows. Finally, some models may work well in some type of interest rate environments, while others do not because of assumptions that must be made. Figure 2.5 graphs the term structure with maturities 3 months, 1 year, and 5 years from 1965 to 2008. In particular, although in the 1980s and 1990s we would think of swaps as derivative securities, which "derive" their price from the value of primary securities, such as Treasuries, it is hard to believe that this is still the case now due to its sheer size. Therefore, the trader is now entitled to receive the repo rate. PREFACE xxvii Chapter 21 introduces a more recent pricing methodology, the forward risk neutral pricing methodology, as well as the more recent Heath, Jarrow, and Morton (HJM) model, and the Brace, Gatarek, and Musiela (BGM) model. Suppose today the interest rate is r = 6%. As it can be seen, the bootstrap method generates a yield curve that has a dip at maturity T = 2.5. It is not clear from the data why the dip in yield is present at that point: It could be a liquidity issue, or staleness, or simply an error in the database. The intuitive answer is 5%, because we invest $100 and obtain $105, and thus the return equals 5% = ($105 − $100)/$100. In Chapter 5 we elaborate on this topic, showing also that a violation of Relation 2.1 amounts to the assumption that future nominal interest rates be negative. The chapter also discusses the important concepts of flat and forward volatility, as well as the dynamics of the term structure of volatility over time. The chapters are highly interrelated and cross-reference each other, and therefore I believe it is pedagogically important to move forward chapter by chapter. In particular, homeowners pay back loans when the interest rate declines. The semi-annual compounding frequency is the standard benchmark, as it matches the frequency of coupon payments of U.S. Treasury notes and bonds. Column (1) reports the time at which coupon payments are made, and rebalancing takes place. Thus, Orange County would have been hit by the same type of losses as if all of its portfolio was invested in zero coupon bonds with 7.9 years to maturity. For instance, in the above example, we implicitly assumed that the 5% rate of interest is applied to the original capital only once (and hence the $105 result). Piecewise polynomial functions, or, spline: Intuitively, a polynomial spline can be thought of as a number of separate polynomial functions, joined smoothly at a number of so called "joints", "breaks," or "knot" points. As time goes by, the time value of money changes. Practically, daily compounding is very close to continuous compounding. The next example illustrates one popular approach to dealing with this latter case. The complexity of fixed income markets is also extraordinary. This number is smaller than the one obtained under the historical distribution approach, because of the fat-tailed distribution of the portfolio P&L, as showin in Figure 3.8: Extreme realizations are more likely under the historical distribution than under the normal distribution. How can we compute the duration of an inverse floater? These fluctuations make their valuation harder. (a) What is its price if its yield to maturity is 6%? EXAMPLE 2.7 Consider the 2-year note issued on t = January 3, 2006 discussed earlier. Look at Table 2.6, which reports all the bond price quotes available on December 31, 1993. Instead of a duration measure, the investor wants now to know the following: (a) What is the dollar duration of each portfolio? The growth in this market is due to the growth of the U.S. real estate market, which boomed in the 2000's to reach its peak in 2006, as well as the steady increase in leverage of U.S. households, who had been taking larger 5 INTRODUCTION Figure 1.1 The Growth in Market Size 9 8 Mortgage Related 7 Corporate Trillion of Dollars 6 5 U.S. Treasury Debt 4 Money Market 3 2 Federal Agency 1 Municipal Asset−Backed 0 1986 1988 1990 1992 1994 1996 1998 2000 2002 2004 2006 2008 Source: The Securities Industry and Financial Markets Association (SIFMA) and larger mortgages and home equity loans to finance consumption. An investor buying this bond in 1993 would have suffered severe capital losses within the next year or so. Since it is a value (what is the value today of $1 in the future), it is essentially a price, describing how much money somebody is willing to pay today in order to have $1 in the future. On top of the $6 trillion Treasury debt, there is a $9 trillion mortgage backed securities market, whose securities (e.g., pass throughs, collateralized debt obligations, and so on) have streams of payments that are not fixed, but depend on various factors, including interest rates' fluctuations. Indeed, the definition of a return on an investment cannot be given without a precise definition of the time interval during which the security is held. I conclude this third part of the book, and the book itself, with Chapter 22, which extends the concepts developed in the previous chapters to the case in which the yield curve is driven by multiple factors. Firm A wants to raise M =$10 million using fixed rate coupon bonds, while Firm B wants to raise M =$10 million using floating rate coupon bonds. Figure 2.1 plot three discounts factors over time, from January 1953 to June 2008. 3.7.1 Benchmark: What if Orange County was Invested in Zero Coupon Bonds Only? In essence, the bank is in trouble. Panel C plots a term structure that is first rising and then decreasing. In the previous section, we computed several numbers in the attempt to measure the risk embedded in the Orange County portfolio. For instance, some models generate simple pricing formulas for relatively complex securities, and this simplicity is useful if a trader needs to compute the prices of a large portfolio of derivatives quickly. We present an example of floating rate bond in Example 2.12. 3.2.5 Properties of Duration It is important to realize that the duration of a coupon bond depends crucially on the level of the coupon rate. Like discount factors, the term spread depends on numerous variables, such as expected future inflation, expected future growth of the economy, agents' attitudes toward risk, and so on. Together with the data sets in such examples, I also include all of the spreadsheets (for Part I and II) or computer codes (for Part III) that generate the results of the analysis in the numerical examples. However, by definition, VaR is concerned with large changes. Instead, the constant strategy of 70% and 30% in long term bond stand to lose money about 10% and 40% of the time, respectively. Exactly because discount factors unambiguously represent a price – an exchange rate between money today versus money tomorrow – they are at the heart of fixed income securities analysis. What is the 95% one-month Value-at-Risk of the portfolio? These interest rates have been simulated. That is, money in hands gives its holder the option to use it however he or she desires, including transferring it to the future through a deposit or an investment. Summary Exercises Appendix: Liquidity and the LIBOR Curve INFLATION, MONETARY POLICY, AND THE FEDERAL FUNDS RATE239 7.1 7.2 7.3 7.4 7.5 The Federal Reserve 7.1.1 Monetary Policy, Economic Growth, and Inflation 7.1.2 The Tools of Monetary Policy 7.1.3 The Federal Funds Rate Predicting the Future Fed Funds Rate 7.2.1 Fed Funds Rate, Inflation and Employment Growth 7.2.2 Long-Term Fed Funds Rate Forecasts 7.2.3 Fed Funds Rate Predictions Using Fed Funds Futures Understanding the Term Structure of Interest Rates 7.3.1 Why Does the Term Structure Slope up in Average? In the following sections we describe their characteristics in more detail. Yet, this portfolio still has a large sensitivity to interest rates, and therefore it is very risky.

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